# A TOPOLOGICAL CLASSIFICATION OF CERTAIN 3-MANIFOLDS

BY G. BURDE AND H. ZIESCHANG

Communicated by Eldon Dyer, May 26, 1967

In this paper we shall prove that a compact 3-manifold, which can be fibered over $S^1$ is topologically determined by its fundamental group and the subgroups belonging to its boundary components.

This theorem was first supposed by J. Stallings [2] and proved in the case of a closed manifold by L. Neuwirth [1]. His proof for bounded orientable manifolds is not complete.

Let $M$ be such a compact 3-manifold with boundary components $B_1, B_2, \cdots, B_r, r \geqq 0$. We denote by $[A_i]$ the class of conjugated subgroups in $\pi_1(M) = G$ generated by loops on $B_i$. We call $\{G, [A_1], \cdots, [A_r]\}$ the peripheral system of $M$.

A fibering of $M$ over $S^1$ is obtained from the product $F \times I$ of a compact surface $F$ and the unit interval $I$ by identifying $(F \times 0)$ and $(F \times 1)$ by a homeomorphism $\zeta$ of $F$. The boundary components of $M$ are tori and Klein-bottles. We write $M = F \times I/\zeta$.

$\zeta$ and $\zeta^*$ belong to the same class $[\zeta]$ of homeomorphisms, if they are connected by the following operations:

(i) isotopic deformation

(ii) conjugation with a homeomorphism of $F$

(iii) replacing $\zeta$ by $\zeta^{-1}$.

$F \times I/\zeta$ and $F \times I/\zeta^*$ are homeomorphic if $\zeta$ and $\zeta^*$ belong to the same class. The proof is immediate (see [1]). We consider the fundamental group $N$ of $F$ as a subgroup of $G$. The projection $M \rightarrow S^1$ induces a homomorphism $\chi: G \rightarrow Z$ with kernel $N$. $\chi(A_i)$, $A_i \in [A_i]$, is a subgroup of $Z$ with finite index $n_i$. $n_i$ is the number of boundary curves of $F$ contained in one boundary component $B_i$ of $M$.[1] $F$ has $n = \sum_{i=1}^{r} n_i$ boundary curves. If $n = 0$, then by $N$ the type of $F$ is given. For $n > 0$, $N$ is a free group, and we have to decide whether $F$ is orientable or not. Let $c_{i0}$ be some boundary curve of $F$ belonging to $B_i$, and choose $A_i \in [A_i]$ such that the generator of the infinite cyclic group $A_i \cap N$ is represented by $c_{i0}$ resp. its inverse $c_{i0}^{-1}$. (The trivial case where $F$ is a disk is excluded.) The cyclic groups $t^j A_i t^{-j} \subset N$, $j = 1, 2, \cdots, n_i - 1$ are generated by the boundary curves $c_{i,j}^{\epsilon_{ij}}$, $j = 1, \cdots, n_i - 1$, $\epsilon_{ij} = \pm 1$, of $F$, contained in $B_i$. The loops $c_{ij}$, $i = 1, \cdots, r$, $j = 0, 1, \cdots, n_i - 1, \cdots, \epsilon_{ij} = \pm 1$, are as elements of $N$ determined modulo conjugation in $N$.

[1] The case $n_i > 1$ was not observed in [1].