INTERMEDIATE EXTENSIONS IN L^p

BY MARTIN SCHECHTER[1]

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1. **Introduction.** Let $A(x, D)$ be an elliptic operator defined on Euclidean n-dimensional space and let $q(x)$ be a locally square integrable function. Let A_0 and B_0 denote the operators $A(x, D)$ and $A(x, D)+q(x)$ acting on the set C_0^∞ of infinitely differentiable functions, respectively. Under suitable regularity conditions on the coefficients of $A(x, D)$ the minimal and maximal closed extension of A_0 in L^p coincide for $1<p<\infty$. Without further restrictions on q, this is not true for B_0.

The purpose of the present investigation is to find sufficient conditions on q such that some closed extension of B_0 will have the same essential spectrum as the closure A of A_0. For $p=2$ we found it convenient in [11] to employ regularly accretive extensions introduced by Kato [13]. However, this theory employs Hilbert space structure and is unapplicable for $p\neq 2$. Moreover, some of the L^2 estimates employed in [11] have no known counterparts in L^p for $p\neq 2$.

Our approach has been to develop a theory of extensions in Banach space which generalizes Kato's development. We call such operators "intermediate extensions." Under suitable conditions on $q(x)$ we are able to show that these extensions have the desired properties.

2. **Intermediate extensions.** Let A_0 be a densely defined, preclosed linear operator from a Banach space X to a Banach space Y. Then $D(A_0^*)$ is weakly* dense in Y^*. Let S be a linear manifold in $D(A_0^*)$ which is also weakly* dense in Y^*. We consider all closed extensions A of A_0 such that $D(A^*)\supseteq S$. The closure $\bar{A}$ of A_0 is the smallest such extension and therefore will be called the *minimal* extension of A_0. There is a largest such extension $\tilde{A}$. $D(\tilde{A})$ consists of those $u\in X$ for which there is an $f\in Y$ satisfying

$$(u, A_0^* v) = (f, v) \qquad \text{for all } v \in S.$$

We then set $\tilde{A}u=f$. This operator is well defined, for if $(u, A_0^*v) = (g, v)$ for all $v\in S$, then $(f-g, v)=0$ for all such v. Since S is weakly* dense in Y^*, we have $f=g$. Moreover, if A is any closed extension of A_0 with $D(A^*)\supseteq S$, then

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