CHERN NUMBERS AND ORIENTED HOMOTOPY TYPE

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1. Statement of results. This note announces the solution of the following problem, a more precise version of which appears in §2:

(1) *Determine the linear combinations of rational Chern numbers (of almost-complex manifolds) that are invariants of oriented homotopy type (of almost-complex manifolds).*

Milnor, who posed the problem, conjectured that every such homotopy invariant could be expressed as a rational linear combination of the Euler Characteristic and the Index.

THEOREM 1. *Milnor's conjecture is true.*

We obtain variations on problem (1) by some simple alterations:

(2) Replace the word "homotopy" in (1) by "diffeomorphism."

THEOREM 2. *Modulo those linear combinations of rational Chern numbers that can be expressed in terms of Pontrjagin numbers, the only combinations that are invariants of oriented diffeomorphism type are the rational multiples of the Euler Characteristic.*

(3) Stabilize (1). That is, consider the same problem with reference to the class of *stably* almost-complex manifolds. According to Milnor (see [2, pp. 122–127]), this does not affect the possible configurations of Chern numbers. However, it does *strictly* decrease the number of independent combinations that are oriented homotopy invariant. Stong has conjectured that multiples of the Index are the only oriented homotopy invariant combinations.

THEOREM 1_S. *Stong's conjecture is true.*

(4) Finally, we may stabilize (2).

THEOREM 2_S. *Every linear combination of rational Chern numbers (of stably almost-complex manifolds) that is an invariant of oriented diffeomorphism type (of stably almost-complex manifolds) can be expressed as a linear combination of rational Pontrjagin numbers.*

We may restate Theorem 2_S in the following way: Let Ω_*^U and Ω_*^{SO} denote the complex and oriented cobordism rings, respectively, and let

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