ON TRANSFORMATIONS OF THE 3-SPHERE FIXING A KNOT

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The *Smith conjecture* (after P. A. Smith [1]) is that no tame, knotted, simple closed curve in the 3-sphere is the fixed point set of a periodic transformation of the 3-sphere. That is to say, for any nontrivial tame knot (S^3, K), the group $F(S^3, K)$ of orientation preserving autohomeomorphisms of S^3 which are the identity on K has no elements of finite order.

The purpose of this paper is to show that, except for the trivial type of knot, the identity path component of $F(S^3, K)$ under the compact-open topology has no elements of finite order. Hence for nontrivial knots, if $F(S^3, K)$ has elements of order $r>1$, then $\pi_0(F(S^3, K))$ has elements of order r.

This suggests a *strong Smith conjecture* that $\pi_0(F(S^3, K))$ has no elements of finite order. The reduction of this strong Smith conjecture to an *algebraic Smith conjecture* about certain automorphisms of the group system of a knot will be the subject of another paper on homeotopy groups of knots.

Indeed, the strong Smith conjecture for torus knots and others may be verified in this manner. This supersedes the special proof of the Smith conjecture for torus knots given by the author in [2]. R. H. Fox [3] has obtained an elementary proof of the Smith conjecture for torus knots and a different, somewhat complicated algebraic interpretation of the Smith conjecture for knots whose knot groups are centerless. The relation of Fox's results with ours will be discussed in the paper on homeotopy groups of knots.

Let (S^3, K) be a nontrivial tame knot, and let

$$h: (S^3, K) \to (S^3, K)$$

be a homeomorphism of pairs such that $h|K=1$, $h^r=1$ for some integer $r>1$, and h is isotopic to the identity (written $h\approx 1$) via an isotopy

$$H: (S^3, K) \times I \to (S^3, K) \times I$$

(i.e. H is a homeomorphism such that $p_I H = p_I$ where p_I denotes the projection of $(S^3, K)\times I$ onto I) such that $H|K\times I=1$. The main theorem is the following.

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