DEMICONTINUITY, HEMICONTINUITY AND MONOTONICITY. II

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In the previous paper [6] with the same title, the writer proved that a (nonlinear) monotonic operator G from a Banach space X to the adjoint space X^* is demicontinuous if and only if it is hemicontinuous and locally bounded, under a certain mild assumption on $D(G)$. (For similar results see also Browder [3].) In the present note we shall show that if $D(G)$ is an open set, the assumption of local boundedness is redundant so that hemicontinuity and demicontinuity are equivalent. Furthermore, we shall show that a similar result holds for a more general class of operators, including *accretive operators* in X where X^* is uniformly convex.

In what follows we consider (real or complex) Banach spaces X, Y and (nonlinear) operators F, G such that (D and R denoting the domain and range, respectively) $D(F)=X$, $R(F)\subset Y$, $D(G)\subset X$, $R(G)\subset Y^*$.

DEFINITION 1. G is said to be F-monotonic if

$$\operatorname{Re}(F(x-y), Gx-Gy) \geqq 0, \qquad x, y \in D(G),$$

where (,) denotes the pairing between Y and Y^*.

DEFINITION 2. Let $u\in D(G)$. G is said to be

(a) demicontinuous at u if $u_n\in D(G)$, $n=1, 2, \cdots$, and $u_n\to u$ as $n\to\infty$ imply $Gu_n\rightharpoonup Gu$ (here and in what follows $\to$ and $\rightharpoonup$ denote strong and weak* convergence, respectively);

(b) locally bounded at u if the conditions in (a) imply that $\|Gu_n\|$ is bounded as $n\to\infty$;

(c) hemicontinuous at u if $v\in X$, $t_n>0$, $n=1, 2, \cdots$, $t_n\to 0$ and $u+t_nv\in D(G)$ imply $G(u+t_nv)\rightharpoonup Gu$;

(d) locally hemibounded at u if the conditions in (c) imply that $\|G(u+t_nv)\|$ is bounded as $n\to\infty$.

Obviously we have the implications

$$(a) \begin{matrix} \nearrow & (b) & \searrow \\ \searrow & (c) & \nearrow \end{matrix} (d).$$

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