LOCAL TIME AT FICTITIOUS STATES

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Theorems 1 and 2 will be contained in a forthcoming paper by the author. Theorem 1 follows from work of Ray [6] and Neveu [4], [5] but a new proof will be presented. Theorem 2 is a consequence of a result already announced in Williams [7]. The terminology and notation are as in Dynkin's book [1].

Let E be a countable set and let $\{p_{ij}(t): i, j \in E; t \geqq 0\}$ be a transition function on E with the properties:

$$\sum_{j \in E} p_{ij}(t) = 1; \qquad \lim_{u \downarrow 0} p_{ii}(u) = 1 \quad (i \in E; t \geqq 0).$$

THEOREM 1. *There exist a complete metric space E^+ in which E is dense and a strong Feller, stochastically continuous transition function $P(t)$ on E^+ such that the following statements are true:*

(i) $$P(t, i, \{j\}) = p_{ij}(t) \qquad (i, j \in E; t \geqq 0);$$

(ii) $$P(t, y, E) = 1 \qquad (y \in E^+; t > 0);$$

(iii) *every Markov chain on E with transition function $\{p_{ij}(t)\}$ has a right-continuous, strong Markov version taking values in E^+ and with transition function $P(t)$.*

For this version, limits from the left may not always exist in E^+ and the property of quasi-left-continuity may not hold.

Let $\{x(t, \omega): t \geqq 0; \omega \in \Omega\}$ be some fixed right-continuous, strong Markov process on E^+ with transition function $P(t)$. Let x be a fixed point of $E^+ \backslash E$ and define

$$\begin{aligned} T(\omega) &= \infty \quad \text{if } x(t, \omega) \neq x \quad \text{for all } t > 0, \\ &= \inf\{t: t > 0, x(t, \omega) = x\} \quad \text{otherwise.} \end{aligned}$$

Suppose that x is such that

(i) $$\boldsymbol{P}_x\{T = 0\} = 1;$$

(ii) $$\boldsymbol{P}_\mu\{T < \infty\} > 0,$$

μ being the initial distribution.[1] Then local time at x may always be defined by an equation similar to equation (1) below.

However, in order that the result may be stated in a form with the

[1] In current terminology, x is neither polar nor semipolar.