# AN APPLICATION OF THE CALCULUS OF VARIATIONS IN THE LARGE TO THE EQUATIONS OF NONLINEAR ELASTICITY[1]

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The partial differential equations associated with many problems in elasticity are distinguished from other equations of mathematical physics by their nonlinearity, ellipticity and variational character. Generally a study of the totality of solutions of the boundary value problem associated with this system of higher order equations is required. The object of this note is to show, by example, that these problems are often mathematically tractable by the combined use of modern regularity theory for elliptic equations and the nonlinear invariants of the associated multiple integral variational problem.

In this note we study the problem of post-buckling behavior of thin elastic structures as defined by Von Karman's equations. In an earlier note [4], the author, together with P. Fife, studied this problem for small deflections of thin plates. Here we outline how this work can be extended to study large deflections of plates and curved elastic structures, (i.e. shells).

Mathematically the associated variational problem requires us to study the critical points of a smooth functional on an infinite dimensional smooth manifold. This manifold arises naturally from the physical problem as a surface of "constant internal energy." This point of view enables us to apply the previous studies of nonlinear elliptic eigenvalue problems [5], [6] and Ljusternik-Schnirelmann category [8], [10] to these problems.

Previous studies of the mathematical aspects of the buckling of thin elastic structures have been studied by Friedrichs [7], Reiss [9], Vorovich [11] among others. The results obtained here are closely related to the numerical studies of Bauer and Reiss [2]; [3]. The author is grateful to Professor E. Reiss for many helpful suggestions in connection with this work as well as suggesting the example of §5.

**1. The boundary value problem and its variational reformulation for a clamped plate.** Let $\Omega$ be a bounded domain in $R^2$ with boundary $\partial\Omega$ consisting of a finite number of arcs on each of which a tangent

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