RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

THE ALGEBRA OF MULTIPLACE VECTOR-VALUED FUNCTIONS[1]

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1. In recent years there has been an upsurge of interest in the algebraic structure of systems of functions (operations, transformations, mappings) closed under various compositions (see, e.g., [1], [2], [4]). We have previously studied one-place functions, with one of our principal results being a complete axiomatic characterization of the semigroup of all partial transformations of a set [5]. In this note we report some of the results of our study of multiplace vector-valued functions. (An outline of a primitive stage of these studies appeared in [6]; a longer paper, with complete proofs and an extended bibliography will appear elsewhere.)

The nature and current state of development of the subject dictate a concrete approach. Accordingly, we work throughout with a given, arbitrary set S and with functions and operations defined in terms of elements of S. In such considerations, it soon becomes apparent that any restriction on the (finite) dimensionality or "fullness" of the domains and ranges of the functions is artificial and obstructive. Consequently, since there is no natural stopping-place, we go all the way, treating the totality of all multiplace vector-valued functions as a whole and admitting all (partial) transformations of Cartesian powers of S into Cartesian powers of S. The resulting system is delineated in §2; the naturalness of the structure may be judged from the results and applications mentioned in this and the two subsequent sections.

2. Let S be a nonempty set. For any positive integer n, let S^n denote the n-fold Cartesian power of S. The elements of S^n will usually be written as strings, e.g., $x_1x_2 \cdots x_n$. Let $\mathfrak{F}_{rd}$ denote the set of all functions whose domain is a nonempty subset of S^d and whose range

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