EQUIVARIANT STABLE STEMS[1]

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Let $S^n(r)$ denote the n-sphere with a linear involution having a fixed point set of codimension r, where $0 \leqq r \leqq n$. We pick some fixed point as a base point and consider the set $[S^n(r); S^k(t)]$ of base point preserving equivariant homotopy classes of maps from $S^n(r)$ to $S^k(t)$. This has a natural group structure for $n-r \geqq 1$ and is abelian if $n-r \geqq 2$.

There is a suspension functor S without action and one Σ with action (that is, the reduced join with $S^1(0)$ and $S^1(1)$ respectively). These induce homomorphisms

$$[S^{n+1}(r); S^{k+1}(t)] \overset{S}{\longleftarrow} [S^n(r); S^k(t)] \overset{\Sigma}{\longrightarrow} [S^{n+1}(r+1); S^{k+1}(t+1)].$$

It can be shown that S is an epimorphism when $n \leqq 2k-1$ and $n-r \leqq 2(k-t)-1$ and is an isomorphism if the strict inequalities hold. Similarly, Σ is an epimorphism when $n \leqq 2k-1$ and $n-r \leqq k-1$ and is an isomorphism if the strict inequalities hold. By passing to the S-limit we define

$$\pi_n(r; t) = \lim_{k} [S^{n+k}(r); S^k(t)].$$

Σ induces $\Sigma: \pi_n(r; t) \to \pi_n(r+1; t+1)$ which is an epimorphism when $n \leqq r-1$ and an isomorphism when $n \leqq r-2$. By passing to the Σ-limit we define

$$\pi_{n,k} = \lim_{t} \pi_n(t+k; t).$$

There is the forgetful functor ψ (forgetting equivariance) and the fixed point set functor ϕ which yield homomorphisms

$$\pi_n \overset{\psi}{\longleftarrow} \pi_n(r; t) \overset{\phi}{\longrightarrow} \pi_{n-r+t} \tag{1}$$

where π_n denotes the classical n-stem. For the doubly stable groups these become

$$\pi_n \overset{\psi}{\longleftarrow} \pi_{n,k} \overset{\phi}{\longrightarrow} \pi_{n-k}. \tag{2}$$

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