EQUIVARIANT COHOMOLOGY THEORIES[1]

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Throughout this note G denotes a discrete group. A G-complex is a CW-complex on which G acts by cellular maps such that the fixed point set of any element of G is a *subcomplex.*

On the category of pairs of G-complexes and equivariant homotopy classes of maps, an *equivariant cohomology theory* is a sequence of contravariant functors $\mathcal{H}^n$ into the category of abelian groups together with natural transformations δ^n: $\mathcal{H}^n(L, \emptyset) \to \mathcal{H}^{n+1}(K, L)$ such that

(1) $\mathcal{H}^n(K \cup L, L) \overset{\approx}{\to} \mathcal{H}^n(K, K \cap L)$ induced by inclusion,

(2) $\cdots \to \mathcal{H}^n(K, L) \to \mathcal{H}^n(K) \to \mathcal{H}^n(L) \to \mathcal{H}^{n+1}(K, L) \to \cdots$ is exact.

(3) If S is a discrete G-set with orbits S_α then

$$\prod_\alpha i_\alpha^*: \mathcal{H}^n(S) \to \prod_\alpha \mathcal{H}^n(S_\alpha)$$

is an isomorphism, where i_α: $S_\alpha \to S$ is the inclusion. (If S/G is finite then (3) follows from the other axioms.)

One should note that, in a sense, the "building blocks" for the G-complexes are the coset spaces G/H and that the equivariant maps $G/H \to G/K$ are also essential data for building G-complexes. Thus we maintain that the "coefficients" of a theory $\mathcal{H}$ should include the groups $\mathcal{H}^n(G/H)$ together with the induced homomorphisms $\mathcal{H}^n(G/K) \to \mathcal{H}^n(G/H)$. We shall make this more precise.

Let $\mathcal{O}_G$ denote the category whose objects are the coset spaces G/H ($H \subset G$) and whose morphisms are the equivariant maps. A *coefficient system* is defined to be a contravariant functor from $\mathcal{O}_G$ to Ab (the category of abelian groups). The coefficient systems themselves form a category $\mathcal{C}_G = [\mathcal{O}_G^*, Ab]$ which is an abelian category with projectives and injectives.

The following remark is useful. For G-sets S and T let $E(S, T)$ denote the set of equivariant maps $S \to T$. Also for $H \subset G$ we let $S^H = \{s \in S \mid h(s) = s \text{ for all } h \in H\}$. The assignment $f \to f(H)$ clearly yields a one-one correspondence

$$E(G/H, S) \overset{\approx}{\to} S^H.$$

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