FIXED POINT FREE INVOLUTIONS ON HOMOTOPY SPHERES

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1. Introduction and statements of theorems. Let $T\colon \Sigma^{n+1}\to\Sigma^{n+1}$ be a smooth[2] (C^∞) fixed point free involution on a smooth manifold, Σ^{n+1}, homeomorphic to the $(n+1)$-sphere, S^{n+1}. We wish to consider the following problem: does there exist an n-sphere, S^n, smoothly imbedded in Σ^{n+1} such that $TS^n=S^n$? If such an S^n exists, we will say that (T, Σ^{n+1}) *desuspends* to $(T|S^n, S^n)$ and that $(T|S^n, S^n)$ *suspends* to (T, Σ^{n+1}). We claim (proofs are to appear later):

THEOREM 1. *If $n\geqq 5$ is odd, then (T, Σ^{n+1}) desuspends to $(T|S^n, S^n)$ for some T-invariant $S^n\subset\Sigma^{n+1}$.*

If n is even, there are obstructions to desuspending (T, Σ^{n+1}). There is a bilinear form, $B(x, y)$ defined on a certain subgroup of $H_*(M)$, where $\Sigma^{n+1}=A\cup TA$, A and TA are compact submanifolds of Σ^{n+1} with smooth boundary, and $\partial A=\partial TA=A\cap TA=M$. If $n\equiv 2 \pmod 4$, then B is symmetric, and its signature, $\sigma(T, \Sigma^{n+1})$ is determined by (T, Σ^{n+1}). If $n\equiv 0 \pmod 4$, then B is skew-symmetric. Furthermore, if $n=4k$, there is a map $\psi_0\colon H_{2k}(M; Z_2)\to Z_2$ such that $\psi_0(x+y)=\psi_0(x)+\psi_0(y)+B_2(x, y)$, where B_2, defined on a subgroup of $H_{2k}(M; Z_2)$, corresponds to B, defined on a subgroup of $H_{2k}(M)$. The Arf invariant, $c(T, \Sigma^{n+1})$, [**1**], [**4**], corresponding to ψ_0 and B_2, depends only on (T, Σ^{n+1}). Regarding these invariants, we have

THEOREM 2. *If $n\equiv 2 \pmod 4$ and $n>5$, then (T, Σ^{n+1}) can be desuspended to $(T|S^n, S^n)$ if and only if $\sigma(T, \Sigma^{n+1})=0$.*

THEOREM 3. *If $n\equiv 0 \pmod 4$ and $n>4$, then (T, Σ^{n+1}) can be desuspended to $(T|S^n, S^n)$ if and only if $c(T, \Sigma^{n+1})=0$.*

At present, we have no example of (T, Σ^{n+1}) for which either $c(T, \Sigma^{n+1})\neq 0$ for $n\equiv 0 \pmod 4$, or $\sigma(T, \Sigma^{n+1})\neq 0$ for $n\equiv 2 \pmod 4$. An interesting example to study in connection with the possibility of

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[2] The results hold equally in the piecewise linear category with little change in the proofs.