HIGHER RANK CLASS GROUPS[1]

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Let A be a noetherian ring which is locally Macaulay. For each integer $i \geqq 0$, groups $C_i(A)$ and $W_i(A)$ are defined, each sequence of groups generalizing to higher dimensions the usual class group of an integrally closed noetherian domain. $C_i(A)$ is called the ith *class group* of A, and $W_i(A)$ is called the ith *homological class group* of A. The main purpose of this note is to show that both sequences of groups have properties analogous to the class group of a Noetherian integrally closed integral domain, and finally to establish a connection between them.

1. Throughout this section A is a commutative noetherian ring which is locally Macaulay. A set of elements $x_1, \cdots, x_s$ is an A-*sequence* of length s if $x_1A + \cdots + x_sA \neq A$ and $x_1A + \cdots + x_iA : x_{i+1} = x_1A + \cdots + x_iA$ for $i = 0, 1, \cdots, s-1$. Count the empty set as an A-sequence of length 0 and specify that it generate the zero ideal of A.

Note that if $x_1, \cdots, x_s$ is an A-sequence of length s, then $x_1A + \cdots + x_sA$ is an unmixed ideal of A of height s.

For each $i \geqq 0$, form the free abelian group on the generators $\langle \mathfrak{p} \rangle$ where $\mathfrak{p}$ is a height i prime ideal of A. This group will be denoted by $D_i(A)$. For each A-sequence $x_1, \cdots, x_i$, consider the element $\sum e(x_1, \cdots, x_i | A_{\mathfrak{p}}) \langle \mathfrak{p} \rangle$ of D_i (here $e(y_1, \cdots, y_i | M)$ denotes the multiplicity of $y_1A + \cdots + y_iA$ on M). Let R_i designate the subgroup of D_i generated by all such elements. Set $C_i(A) = D_i(A)/R_i$ and call $C_i(A)$ the *class group of rank i* for A. Denote the image of $\langle \mathfrak{p} \rangle$ in $C_i(A)$ by $\mathrm{cl}(\mathfrak{p})$. Set $C.(A) = \oplus C_i(A)$.

EXAMPLES. $C_0(A)$ is always finitely generated. $C_0(A)$ is finite if and only if (0) is a primary ideal of A. $C_0(A) = 0$ if and only if A is a domain.

If A is a Dedekind domain, then $C_1(A)$ is the ordinary ideal class group of A. More generally, if A is integrally closed, then $C_1(A)$ is the class group of A [1, §1, no. 10].

We have not been able to locate the following lemma in the literature.

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