AUTOMORPHISMS OF COMPACT RIEMANN SURFACES AND THE VANISHING OF THETA CONSTANTS

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I. It is the purpose of this note to announce a theorem which shows that there exists a connection between automorphisms of compact Riemann surfaces and the vanishing of Riemann theta constants. In particular we shall outline the proof of the following theorem:

THEOREM 1. *Let S be a compact Riemann surface of genus $2g-1$, $g\geqq 2$, which permits a conformal fixed point free involution T. Let $\gamma_1, \cdots, \gamma_{2g-1}; \delta_1, \cdots, \delta_{2g-1}$ be a canonical dissection of S and let T be such that $T(\gamma_1)$ is homologous to γ_1, $T(\delta_1)$ is homologous to δ_1, $T(\gamma_i)$ is homologous to γ_{g+i-1} and $T(\delta_i)$ is homologous to δ_{g+i-1}, $i=2, \cdots, g$. Then, there exist at least $2^{g-2}(2^{g-1}-1)$ half integer theta characteristics $\epsilon_1, \cdots$ such that $\theta_{\epsilon_1}(0)=\theta_{\epsilon_2}(0)= \cdots =0$ and the order of the zero is $\geqq 2$.*

The proof of the theorem rests on the fact that S is a two sheeted nonbranched covering of a compact Riemann surface of genus g and the following lemma which was proved in [1].

LEMMA 1. *Let ζ,ω be equivalent special divisors of degree $G-1$ on a compact Riemann surface S of genus G. Then if $i(\zeta\omega)=1$, where "i" is the index of specialty of the divisor, there exists a half integer characteristic ϵ corresponding to the divisor ζ such that $\theta_\epsilon(0)=0$ and the order of the zero is $\geqq 2$. θ is of course the Riemann theta of S.*

II. Let $\hat{S}=S/T$ denote the compact Riemann surface of genus g which is covered by S. Then all the functions and differentials which exist and are well defined on $\hat{S}$ may be lifted to S and are well-defined objects thereon. As a matter of fact all such lifted functions and differentials will be invariant under the involution T and conversely all objects on S which are invariant under T are well defined on $\hat{S}$. There are, however, objects which are not well defined on $\hat{S}$ but are well defined on S. For example, let $\hat{\theta}_\alpha$ and $\hat{\theta}_\beta$ be two odd Riemann thetas associated with $\hat{S}$ such that $\alpha+\beta\equiv\delta$ where δ is the characteristic $(0, \cdots, 0; \frac{1}{2}, 0 \cdots 0)$, α, β half integer characteristics. Then the quotient $\hat{\theta}_\alpha/\hat{\theta}_\beta$ is not well defined on $\hat{S}$ for analytic continuation of

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