LOCALLY COMPACT TRANSFORMATION GROUPS AND C^*-ALGEBRAS

BY EDWARD G. EFFROS[1] AND FRANK HAHN[2]

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It has long been recognized that one may associate operator algebras with transformation groups (see, e.g. [**9**, Chapter III], [**11**; 1, p. 310] [**5**]). In this paper we shall answer two questions about the ergodic invariant probability measures on a locally compact transformation group (Theorems 2 and 3). This information is then used to solve analogous problems for the unit traces of a C^*-algebra (Theorems 5 and 6). Full proofs will appear elsewhere.

Let (G, Z) be a topological transformation group with G and Z second countable and Hausdorff, G a locally compact group, and Z a compact space. Let Z/G be the set of orbits $G\zeta$ with ζ in Z, together with the quotient topology. Define an equivalence relation $\sim$ on Z/G by $p\sim q$ if the sets $\{p\}$ and $\{q\}$ have the same closure, and let $(Z/G)^{\sim}$ be the equivalence classes with the quotient topology (see [**8**, p. 58]). The elements of $(Z/G)^{\sim}$ are in one-to-one correspondence with the subsets of Z that are closures of orbits. Z/G is T_0 if and only if $\sim$ is trivial, and T_1 if and only if the orbits are closed.

Let Z be compact and let $C(Z)$ be the continuous complex valued functions on Z with the uniform norm, and $M(Z)=C_r(Z)^*$ the real Radon measures on Z with the weak* topology. Let G act on $C(Z)$ and $M(Z)$ by translation, i.e., for s in G, ζ in Z, f in $C(Z)$, and μ in $M(Z)$, let

$$(sf)(\zeta) = f(s^{-1}\zeta),$$
$$(s\mu)(f) = \mu(s^{-1}f).$$

Let $M_G(Z)$ be the invariant measures on Z, and $P_G(Z)$ the corresponding probability measures, i.e.,

$$P_G(Z) = M_G^+(Z) \cap H,$$

where $M_G^+(Z)$ are the positive invariant measures, and H are the measures μ such that $\mu(Z)=1$. $P_G(Z)$ is a compact simplex in the sense of Choquet, and its extremal points are just the ergodic mea-

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[2] National Science Foundation Fellow.