THE MATRIX GROUP OF TWO PERMUTATION GROUPS[1]

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1. **Introduction.** In his remarkable combinatorial paper, Redfield [3] derived a formula for the number of equivalence classes of "range correspondences." It can be obtained by applying Burnside's well-known theorem [1] for the number of orbits determined by a permutation group and is used to enumerate superposed graphs [2]. In this announcement we construct a more general permutation group, called the "matrix group" and give an explicit expression for the number of orbits it determines. This result enables us to enumerate superposed graphs composed of interchangeable copies of the same graph.

2. **The matrix group.** Let A be a permutation group of degree m acting on the set $X=\{1, 2, \cdots, m\}$. For any permutation α in A, we denote by $j_k(\alpha)$ the number of cycles of length k in the disjoint cycle decomposition of α. Let $a_1, a_2, \cdots, a_m$ be variables. The cycle index $Z(A)$ is given by

$$Z(A) = \frac{1}{|A|} \sum_{\alpha\in A} \prod_{k=1}^{m} a_k^{j_k(\alpha)} . \tag{1}$$

It is often convenient to write $Z(A)=Z(A; a_1, a_2, \cdots, a_m)$ to indicate the variables used.

Now let B be another permutation group of degree n acting on $Y=\{1, 2, \cdots, n\}$. Let W be the collection of m by n matrices in which the elements of each row are the n objects in Y. Thus there are $(n!)^m$ matrices in W. Two matrices in W are called *column-equivalent* if one can be obtained from the other by a permutation of the columns. Hence there are $(n!)^{m-1}$ column-equivalence classes.

The *matrix group* of A and B, denoted $[A; B]$, acts on the column-equivalence classes as follows. For each permutation α in A and each sequence $\beta_1, \beta_2, \cdots, \beta_m$ of m permutations with β_i in B, there is a permutation, denoted $[\alpha; \beta_1, \beta_2, \cdots, \beta_m]$ in $[A; B]$ such that the column-equivalence class to which the matrix $[w_{i,j}]$ belongs is sent by $[\alpha; \beta_1, \beta_2, \cdots, \beta_m]$ to the class to which $[\beta_i w_{\alpha i,j}]$ belongs. That is, α first determines a permutation of the rows and then each β_i permutes the entries in the ith row.

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