PIECEWISE LINEAR TRANSVERSALITY

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We prove transversality theorems for piecewise linear manifolds, maps and polyhedra. Our main result is that given two closed manifolds contained in a third, then one can be ambient isotoped until it is transversal to the other. This result is then extended to maps and polyhedra.

The transversality theory for smooth manifolds was initiated by Thom in his classical paper [8], and has been extended to piecewise linear manifolds by Williamson [9]. For both of these authors the raison d'être was cobordism theory, and the transversality theorem that was needed was the following: given a map $M \to Q$ between manifolds then it can be homotoped transversal to a given submanifold P of Q. In order to prove this P was assumed to have a normal bundle, and the technique was to slide the map locally along the fibres, and then globalise by using Baire's theorem in the function space.

However, as yet the existence of normal bundles in the piecewise linear category is an open question. Haefliger and Wall [3] have shown that normal bundles exist in the stable range, but Hirsch [5] has shown that normal disc bundles do not always exist in the unstable range, and this suggests that normal bundles may not exist either. To cope with this difficulty Rourke and Sanderson [7] have recently introduced block bundles, which differ from ordinary bundles by having a block over each simplex instead of a disc over each point. Similar theories have been introduced by Haefliger [4] and Morlet [6]. The importance of block bundles is that in the piecewise linear category normal block bundles exist and are unique. With this tool Rourke and Sanderson [7, II] have proved a transversality theorem similar to Theorem 1 below. Like ours it is an isotopy theorem, unlike the homotopy theorem of Williamson mentioned above. Like us, they use direct geometric methods rather than function space methods, because in the function space of embeddings, those that are transversal do not form an open set.

When generalising to polyhedra, block bundles are no good because the regular neighbourhood of a polyhedron in a manifold is not a block bundle. The technique that we use deals with both submanifolds and subpolyhedra. The idea is to triangulate the ambient manifold so that one object is a subcomplex, and then ambient isotope the other so that it cuts across the triangulation—we call this *transimplicial*. As a result the two objects will then be transversal.

We now confine ourselves to definitions of transversality and state-