ON LIE ALGEBRAS OF TYPE E_6

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Communicated by G. D. Mostow, September 21, 1966.

Introduction. In this note, we investigate, omitting details, the structure of Lie algebras of type E_6 over arbitrary fields of characteristic other than two or three, introducing certain invariants for such algebras and studying the implications these invariants have for the structure of the algebras in question. As a consequence of this investigation, we obtain, producing constructively a representative of each isomorphism class, a complete classification of algebras of type E_6 over finite, real closed, or p-adic fields, as well as partial results for algebraic number fields. Since every Lie algebra of type E_6 over Φ has a finite, Galois, splitting field $P \supseteq \Phi$, [1], we restrict our attention, without loss of generality, to a particular pair of fields P and Φ, P finite, Galois over Φ with group G, and to the collection of Lie algebras of type E_6 over Φ which are split by P.

Realization of the split E_6. Let $\mathfrak{J}_0$ be a split exceptional central simple Jordan algebra over Φ, $\mathfrak{J} = \mathfrak{J}_0 \otimes_\Phi P$, and V the P-space of all

$$x = \begin{pmatrix} \alpha_1 & a_1 \\ a_2 & \alpha_2 \end{pmatrix}, \ \alpha_i \in P, \ a_i \in \mathfrak{J}.$$

V, with quartic form

$$q(x) = 8(a_1 \times a_1, a_2 \times a_2) - 8\alpha_1 N(a_1) - 8\alpha_2 N(a_2) - 2((a_1, a_2) - \alpha_1\alpha_2)^2,$$

(a, b) the trace bilinear form of $\mathfrak{J}$, $N(a)$ the generic norm on $\mathfrak{J}$, $\times$ the product defined in [4], is a module for the split Lie algebra of type E_7 [4], [8]. The algebra $\mathfrak{L}(V, V_0) = \{L \in \mathrm{Hom}(V, V) \mid V_0 L = 0,\ L$ skew with respect to the linearized $q(x)\}$, V_0 the subspace of V of diagonal elements, is a split Lie algebra of type E_6. The semiautomorphisms of $\mathfrak{L}(V, V_0)$ are described by

THEOREM 1. *$A(s)$ is an s-semiautomorphism of $\mathfrak{L}(V, V_0)$ if and only if there is a permutation π of $\{1, 2\}$, an element $\gamma \in P^*$, and an s-semilinear transformation $T(s)$ on $\mathfrak{J}$ satisfying $N(xT(s)) = \mu N(x)^s$ for all $x \in \mathfrak{J}$, such that*

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[1] The contents of this paper form part of the author's dissertation at Yale University, written under the direction of Professor N. Jacobson while the author was a National Science Foundation Graduate Fellow.