SCHOTTKY GROUPS AND LIMITS OF KLEINIAN GROUPS[1]

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DEFINITION. A group of Möbius transformations is said to be a *marked Schottky group* of genus g with *standard generators* $T_1, \cdots, T_g$, if there exist disjoint Jordan curves $\Gamma_1, \Gamma_1', \cdots, \Gamma_g, \Gamma_g'$, which bound a $2g$-times connected domain D, such that $T_j(D) \cap D = \emptyset$ and $T_j(\Gamma_j) = \Gamma_j', j = 1, \cdots, g$.

LEMMA 1. *If G is a Schottky group of genus g, then G is a marked Schottky group on every set of g free generators; i.e., every set of g free generators for G is standard.*

The proof is based on the classical theorem on automorphisms of finitely generated free groups.

LEMMA 2. *Every finitely generated subgroup of a Schottky group is a Schottky group.*

The proof uses the fact that all Schottky groups are quasi-conformally equivalent to certain Fuchsian groups.

REMARK. The preceding two lemmas can be generalized, with appropriate modifications, to *Schottky type groups* (see [1] for the definition).

THEOREM 1. *Let* $T_1, \cdots, T_g$ *be* $g > 1$ *Möbius transformations. Suppose there exist marked Schottky groups of genus g,* $\langle T_{1,n}, \cdots, T_{g,n} \rangle$, *such that* $\lim_{n\to\infty} T_{j,n} = T_j$, $j = 1, \cdots, g$. *Then the group G generated by* $T_1, \cdots, T_g$ *is a free group on g generators, without elliptic elements.*

The proof uses Lemmas 1 and 2 and involves an elementary area argument.

DEFINITION. An isomorphism $\phi: G_1 \to G_2$ between two Kleinian groups is said to be *type preserving* if $\phi(T)$ is parabolic if and only if T is.

THEOREM 2. *For every* $n = 0, 1, 2, \cdots$, *let* $G(n) = \{T_j(n), j = 0, 1, \cdots\}$ *be a Kleinian group. Assume that there are Möbius transformations* T_j *such that* $\lim_{n\to\infty} T_j(n) = T_j$, *and denote the group* $\{T_j, j = 0, 1, \cdots\}$ *by G. Assume also that all mappings* $T_j(0) \to T_j(n)$

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