THE RADIAL HEAT EQUATION WITH POLE TYPE DATA[1]

BY L. R. BRAGG

Communicated by A. Zygmund, August 22, 1966

1. **Introduction.** Recently, detailed studies have been undertaken relating to the solutions and expansions of solutions of the initial value problem

$$(1)\qquad \begin{aligned} &\text{(a)}\quad U_t(r, t) = \Delta_\mu U(r, t), \qquad r > 0,\ t > 0,\\ &\text{(b)}\quad U(r, 0) = \phi(r)\end{aligned}$$

with $\Delta_\mu \equiv D_r^2 + [(\mu-1)/r]D_r$. Results have been obtained when $\phi(r)$ is entire of growth $(1, \sigma)$ in r^2 [1], [3], [4] and these have been extended to the L_2 theory in [3]. In this note, we state some results on the structures of solutions of (1) when the data function $\phi(r)$ has a pole at $r=0$ but is otherwise entire. These structures are defined in terms of convolution integrals and the proofs are based on the Laplace transform formulation [2] of solutions of (1) and the expansion theory referred to above. The details of the proofs will appear in a forthcoming paper that will also discuss logarithmic singularities.

We denote by $U^\mu(r, t; \phi(r))$ the solution of (1) defined by

$$\int_0^\infty K_\mu(r, \xi; t)\phi(\xi)d\xi$$

with

$$K_\mu(r, \xi; t) = \frac{1}{2t} r^{1-\mu/2}\xi^{\mu/2} \exp\left[-(r^2 + \xi^2)/4t\right] I_{\mu/2-1}(r\xi/2t).$$

(See [1], [4].) The abbreviation $a = r^2/16t^2$ will be used in the statement of results.

2. **Main results.** Our first theorem relates to functions $\phi(r)$ that are odd while the remaining results relate strictly to functions with poles.

THEOREM 1. *Let $\phi(r) = r\psi(r)$ in which $\psi(r)$ is an entire function of r^2 of growth $(1, \sigma)$. For $0 \leqq t < 1/4\sigma$ and $\mu > 2$,*

[1] This research was supported by the National Aeronautics and Space Administration Grant No. NsG-544.