COCOMMUTATIVE HOPF ALGEBRAS WITH ANTIPODE

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Communicated by A. Mattuck, June 1, 1966

We shall describe the structure of a certain kind of Hopf algebra over an algebraically closed field k of characteristic p, namely those Hopf algebras whose coalgebra structure is commutative and which have an antipodal map $S: H \to H$. (See below for definitions.) Such a Hopf algebra turns out to be of the form $kG \# U$, the smash product of a group algebra with a Hopf algebra whose coalgebra structure is "like" that of a universal enveloping algebra. If $p=0$ the second factor actually is a universal enveloping algebra.

For $p>0$, we generalize the Birkhoff-Witt theorem by introducing the notion of divided powers. These also play a role in the theory of algebraic groups where certain sequences of divided powers correspond to one parameter subgroups. The divided powers appear in a "Galois Theory" for all finite normal field extensions.

The structure theory of Z_2-graded coanticommutative Hopf algebras is similar, and mentioned below.

Lemma 1, Theorem 1, its generalization to the graded case, and Theorem 2 are unpublished results of B. Kostant, whose guidance we gratefully acknowledge.

1. H is a cocommutative Hopf algebra with multiplication m, augmentation ϵ and diagonal d.

DEFINITION. An element $g \in H$ is *grouplike* if $dg = g \otimes g$ and $g \neq 0$.

LEMMA 1. *The set G of grouplike elements of H form a multiplicative semigroup whose elements are linearly independent in H. For each $g \in G$ there exists a unique maximal coalgebra $H^g \subset H$ whose only grouplike element is g. $H \cong \oplus H^g$ as a coalgebra, and $H^g H^h \subset H^{gh}$.*

DEFINITION. $S: H \to H$ is an *antipode* if

$$m \circ (I \otimes S) \circ d = \epsilon = m \circ (S \otimes I) \circ d.$$

THEOREM 1. *If H has an antipode G is a group and $S(g) = g^{-1}$. If e is the identity of G, $H^g = gH^e = H^e g$, and $H \cong kG \# H^e$ as a Hopf algebra.*

REMARK. Since $g^{-1}H^e g = H^e$, the elements of G act as Hopf algebra automorphisms of H^e and so we can form the smash product $kG \# H^e$.

[1] Part of the research described here was done while the author held an N.S.F. Graduate Fellowship.