HEREDITARILY RETRACEABLE ISOLS[1]

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In [3], it was shown that the recursive isomorphism type of a co-r.e. retraceable set may contain nonretraceable sets. The question naturally arises whether *every* nonrecursive, co-r.e. retraceable set gives rise, under some recursive permutation, to a nonretraceable set. It is an obvious corollary to the first theorem announced in this note that the answer is "no."

If an infinite retraceable set has no nonretraceable regressive subsets, we term it *hereditarily retraceable*; a *hereditarily retraceable isomorphism type* is then a recursive isomorphism type consisting exclusively of such sets. Likewise, a *hereditarily retraceable isol* is a recursive equivalence type each member of which is an immune retraceable set having no nonretraceable regressive subsets. Such, then, are the objects referred to in the title of the note; the existence of a continuum of them follows from Theorem 1 below. Our terminology is, in all other respects, that of [1], [2].

THEOREM 1. *If α is an infinite retraceable set, then α has an infinite retraceable subset β with the following property:*

$(\forall f)$*(f a one-one partial recursive function$\Rightarrow$every regressive subset of $f(\beta)$ is retraceable.)*

Moreover, if α has recursively enumerable complement then we can satisfy the additional requirement that β have recursively enumerable complement.

The proof, which will appear in detail elsewhere, is accomplished by means of a simple priority scheme which (a) applies to any retracing function, and (b) is designed to exploit the following lemma (the truth of the lemma is obvious):

LEMMA 0. *If an infinite set β of natural numbers is regressed by the partial recursive function f, then either β is retraceable or there are infinitely many numbers $b \in \beta$ such that $f(b) > b$.*

REMARK 1.5. With regard to the analogy

$$\frac{\text{recursive}}{\text{r.e.}} = \frac{\text{retraceable}}{\text{regressive}}$$

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