# COHOMOLOGY OF ALGEBRAIC GROUPS AND INVARIANT SPLITTING OF ALGEBRAS[1,2]

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1. **Introduction.** Let $A$ be an algebra, over a field $F$, assumed at first to be associative and finite-dimensional over $F$. Let $R$ be the radical of $A$, $C$ the center of $A$. Assume $A/R$ separable, so that $A$ possesses maximal separable subalgebras (Wedderburn factors) $S$ for which $A=S+R$, $S\cap R=0$. Let $G$ be a group of automorphisms and antiautomorphisms of $A$. We will discuss the existence and uniqueness of $G$-invariant Wedderburn factors in terms of various cohomology groups of $G$. In general, the cohomology is that of abstract groups. However, the conditions given will be compatible with taking the algebraic hull of $G$ (in the Zariski topology with respect to $F$), so that we can assume $G$ is an algebraic group and the cohomology is rational. We will outline here how the cohomology enters. Details will appear elsewhere. See [3], [4], [5] for a general background of the question.

2. **Existence.** We first assume $R^2=0$. Let $S$ be any maximal separable subalgebra. If $g\in G$, then $Sg$ is another maximal separable subalgebra, so by the Malcev theorem, $Sg=SC_{1-z(g)}$, where $C_w$ is conjugation by $w$. $z(g)$ is in $R$, but is uniquely determined modulo $R\cap C$, so that we consider $z$ as a function from $G$ to the vector space $R/R\cap C$. We consider $R/R\cap C$ as a $G$-module in the obvious way, except that the antiautomorphisms in $G$ act via their negatives. Then a technical calculation will show that $z\in Z^1(G, R/R\cap C)$, i.e., $z(gh)=z(g)\cdot h+z(h)$. Hence if $H^1(G, R/R\cap C)=0$, there is an $x$ in $R$ such that $z(g)=x-x\cdot g+R\cap C$. A technical calculation will then show that $SC_{1-x}$ is a $G$-invariant maximal separable subalgebra.

Now we consider the general case $R^2\neq 0$. The action of $G$ on all modules will be the obvious ones, except that the antiautomorphisms in $G$ will act via their negatives. We consider $A/R^2$. The condition for the case $R^2=0$ above now becomes $H^1(G, R/\{x\in R\mid [A,x]\subseteq R^2\})=0$ where $[A, x]=\{[a, x]=ax-xa \mid a\in A\}$. If this holds, then $A=S_1+R$, $S_1$ a $G$-invariant subalgebra, $S_1\cap R\subseteq R^2$. $S_1$ has radical $R^2$, and we next consider $S_1/R^4$. The condition now is

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