ON LATTICE-POINTS IN A RANDOM SPHERE

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1. I. M. Vinogradov and A. G. Postnikov, in their one-hour report on *Recent developments in analytic number theory* at the International Congress of Mathematicians (Moscow, 1966) have referred to a recent result of A. A. Judin on the lattice-point problem for a random circle. If (α, β) is an arbitrary point in the plane, and $A(x; \alpha, \beta)$ denotes the number of lattice-points inside and on the circumference of a circle with (α, β) as centre and $x^{1/2}$ as radius, then Judin's result, as stated in the above-mentioned report, is that

$$\limsup_{x\to\infty} \frac{|A(x; \alpha, \beta) - \pi x|}{x^{1/4}} > c > 0,$$

and the proof, according to the report, is by the application of arguments from the theory of almost periodic functions. This is of interest in view of the known result [3] that

$$A(x; \alpha, \beta) - \pi x = O(x^{1/4+\epsilon}), \quad \epsilon > 0,$$

for almost all points (α, β).

It is our object to show that the following result, hence also Judin's, is a direct consequence of a theorem of ours on the average order of arithmetical functions:

$$\limsup_{x\to\infty} \frac{A(x; \alpha, \beta) - \pi x}{x^{1/4}} > 0,$$

$$\liminf_{x\to\infty} \frac{A(x; \alpha, \beta) - \pi x}{x^{1/4}} < 0.$$

This result is true not only in the plane, but in k dimensions, for $k \geqq 2$. Instead of $A(x; \alpha, \beta)$, one can also consider its higher averages of order $\rho \geqq 0$, the proof being the same.

2. THEOREM. *If $(\alpha_1, \cdots, \alpha_k)$ is an arbitrary point in k-space, $k \geqq 2$, and $A(x; \alpha_1, \cdots, \alpha_k)$ denotes the number of lattice-points inside and on a sphere with centre $(\alpha_1, \cdots, \alpha_k)$, and radius $x^{1/2}$, then*

$$[A(x; \alpha_1, \cdots, \alpha_k) - \pi^{k/2}x^{k/2}/\Gamma(k/2+1)] = \Omega_{\pm}(x^{(k-1)/4}),$$

as $x \to \infty$.