AN ALGEBRAIC CHARACTERISATION OF CONVERGENCE IDEALS

BY JANE M. O. SPEAKMAN[1]

Communicated by M. Loève, August 25, 1966

We consider series whose terms are nonnegative real numbers. The terms of a series will be indexed by Z, the set of positive integers. Where there is no ambiguity we shall write $\sum_A$ for $\sum_{n\in A} a_n$ where A is a subset of Z. The notation $A+B$ where A and B are sets will be used only when A and B are disjoint; it represents the union of A and B. Even if $\sum a_n$ diverges there will be a large number of convergent subseries and the class $\mathcal{I}$ of all the corresponding subsets of Z forms an *ideal* (the convergence ideal). This means that (i) $A\in\mathcal{I}$ and $B\subset A\Rightarrow B\in\mathcal{I}$, (ii) A and $B\in\mathcal{I}\Rightarrow A\cup B\in\mathcal{I}$ and (iii) $\emptyset\in\mathcal{I}$. Kakutani [1] has given a necessary and sufficient condition for two series to give rise to the same convergence ideal, but the problem of characterising convergence ideals appears to have been open until now, although some necessary and some sufficient conditions for an ideal to be a convergence ideal have been found by N. G. de Bruijn, P. Erdös, S. Kakutani and R. Rado (unpublished). This note describes a necessary *and* sufficient condition for an ideal to be a convergence ideal; this will be based on the new concept of a *portability-class*. A complete account of the work will be published elsewhere.[2]

DEFINITION. A class $\mathcal{P}$ of *finite* subsets of Z is a portability-class if and only if it obeys rules (1°)–(6°) below. The elements of the class will be called the portable sets.

(1°) *All one-point sets are portable.*

(2°) *All subsets of portable sets are portable.*

If A and B are portable sets and D is a finite set disjoint from A and B such that $A+D$ is portable while $B+D$ is not, then

(3°) *for all finite sets E disjoint from A and B either $A+E$ is portable or $B+E$ is not, and*

(4°) *given any finite set C there exists a finite set F disjoint from $A\cup B\cup C$ such that $A+F$ is portable while $B+F$ is not.*

Under (1°)–(4°) we can define a relation on the portable sets by saying that A is *not heavier* than B, written $A\leqq B$, if and only if, for

[1] This work was done during the tenure of a Science Research Council Studentship and a research studentship from Girton College.

[2] The paper is to appear in the Journal of the London Mathematical Society.