SOME CLASSICAL THEOREMS ON OPEN RIEMANN SURFACES[1]

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There are many approaches to the study of open Riemann surfaces. I shall mention only three of these. First, one can ask how to generalize the classical theory of compact Riemann surfaces; that is, the theory of algebraic curves over the complex numbers. My talk will be concerned with this approach. Secondly, one can ask how much of the classical theory of meromorphic functions in the plane or unit disk carries over to more general domains. I shall not be concerned with this problem today although perhaps this is a more reasonable approach than the first, since open surfaces do not really seem a proper object for algebraic investigation. (This, however, will not prevent me from speaking on the subject.) Thirdly, one can deal with the problem of classification of surfaces. This topic is, I think, almost unavoidable in any discussion of open surfaces since it is difficult to make general statements which do not trivialize for some important class of surfaces. This will be particularly true for theorems with algebraic origins, although there are notable exceptions. Theorems concerning periods of differentials will make little sense in the context, say, of the unit disk. Consequently, I shall have to discuss to some extent the classification problem in order that you understand the types of surfaces where one can reasonably hope for analogues of theorems from classical algebraic geometry.

The classical theorems I want to discuss are the following: Abel's theorem, the Riemann-Roch theorem, and the theorem of Torelli. Let me remind you of the classical theorems in a form that seems most easily generalized. The classical theory may be said to start with the observation that the only functions meromorphic on the Riemann sphere are the rational functions. On the Riemann sphere we may prescribe the zeros and poles of a rational function subject only to the restriction that the numbers of zeros and poles be the same if we adopt the usual conventions when counting multiple values.

If one considers the field of meromorphic functions on a compact Riemann surface, then, algebraically, this field is a finite (algebraic)

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