DUALITY IN A TWISTED HOMOLOGY THEORY

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In this note we give definitions of twisted generalized homology and cohomology, in which the twisting is defined by spherical fibrations, and we prove that under certain conditions Poincaré duality holds. The definitions are chosen such that when the fibrations are trivial, Whitehead's definitions, as given in [6], hold good. We use the notation of [6] and in the future refer to twisted generalized homology and cohomology simply as homology and cohomology and to spherical fibrations as fibrations. Full details of the work reported here will appear elsewhere.

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1. Let X be a finite connected C.W. complex with base point and denote by Λ the group ring of $\pi_1(X)$. Suppose (i) we have a homomorphism $w: \pi_1(X) \to \{\pm 1\}$ defining a Λ-module structure $\boldsymbol{Z}^t$ on $\boldsymbol{Z}$ and (ii) there is an integer n and a class $[X] \in H_n(X; \boldsymbol{Z}^t)$ such that, for all r, cap product with $[X]$ induces an isomorphism

$$[X]\cap: H^r(X; \Lambda) \to H_{n-r}(X; \Lambda \otimes \boldsymbol{Z}^t)$$

then we say X is an *n-dimensional connected Poincaré complex*. By a space X, we shall mean a connected Poincaré complex. We assume that for all spectra [6] $\mathfrak{A}$ mentioned, there exists an integer N such that A_{N+i} is i-connected for all $i \geqq 0$ and we denote by $e(A)$ the map $SA_n \to A_{n+1}$ (all n) where SA_n denotes (reduced) suspension of A_n, $S^1 \wedge A_n$.

Let ξ be an $(r-1)$ spherical fibration [2] over a space X and denote by X^ξ its Thom space (mapping cone of the projection map). Then we define the pth cohomology group of X, with values in a spectrum $\mathfrak{A}$ twisted by ξ, as

$$H^p(X; \mathfrak{A}(\xi)) = \varinjlim_{\alpha} \{S^\alpha X^\xi, A_{\alpha+r+p}\}.$$

$\{V, W\}$ denotes stable homotopy classes of S-maps of V into W; the direct limit is over the positive integers and the connecting homomorphisms are given by suspension composed with $e(A)$. (As all our groups are reduced we write $H^p(X)$ for $H^p(X, x_0)$.) If η is an $(s-1)$-

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