ON THE HAUPTVERMUTUNG, TRIANGULATION OF MANIFOLDS, AND h-COBORDISM

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We consider the question of uniqueness and existence of piecewise linear structures on manifolds.

I. **Some relations between existence and uniqueness.** By a manifold we will, in general, mean a topological manifold with or without boundary, compact or not. A PL manifold will be a topological manifold along with a given triangulation as a combinatorial manifold. A PL map will be the usual thing. If M is a manifold, $t(M)$ will denote its topological tangent bundle [1]. A tangential equivalence $f: M \to M'$ will be a homotopy equivalence such that $t(M)$ and $f^*t(M')$ are stably equivalent. An h-cobordism, W, will be a compact manifold with $\partial W = \partial_0 W \cup \partial_1 W$, where $\partial_i W$ are the components of ∂W such that there exists a manifold M and a homotopy equivalence

$$f\colon (W, \partial_0 W \cup \partial_1 W) \to (M \times I, M \times (0) \cup M \times (1)).$$

$\partial_0 W$ and $\partial_1 W$ are said to be h-cobordant. $[X, Y]$ will denote the set of homotopy classes of maps.

DEFINITION. *The closed manifold M satisfies condition α_n^k if*

(a) $\dim M \geqq k$

(b) *M is n-connected if $n > 0$, $\pi_1(M)$ is free abelian and finitely generated if $n = 0$.*

Consider the following statements:

A_n^k—Every closed manifold satisfying α_n^k is homeomorphic to a PL manifold.

B_n^k—If M^1, M^2 satisfy α_n^k and M^1, M^2 are h-cobordant then M^1 is homeomorphic to M^2.

C_n—For each n-connected closed manifold M, there exists an l such that $M \times R^l$ is homeomorphic to a PL manifold.

THEOREM A. $A_n^k \Leftrightarrow B_n^k + C_n$.

Now consider the statement:

D_n^k—If M^1, M^2 are simply connected PL manifolds satisfying α_n^k and if M^1, M^2 are h-cobordant as topological manifolds, then there exists a PL isomorphism between M^1, M^2.

THEOREM B. $C_n \Rightarrow D_n^k$.