EXTREMELY AMENABLE SEMIGROUPS[1]

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Let S be a discrete semigroup and $m(S)$ the Banach space of bounded real functions on S with the usual norm $\|f\| = \sup\{|f(s)|; s \in S\}$. A linear functional ϕ on $m(S)$ is a mean if $\phi(f) \geqq 0$ for $f \in m(S)$ with $f \geqq 0$ and $\phi(1) = 1$, where 1 is the constant one function on S.

S is *extremely left amenable* (ELA) if there exists a *multiplicative* left invariant mean ϕ on $m(S)$, i.e. a mean ϕ which satisfies $\phi(fg) = \phi(f)\phi(g)$ for any $f, g \in m(S)$ and $\phi(f_a) = \phi(f)$ for each $f \in m(S)$ and $a \in S$ (where $f_a(s) = f(as)$ and $f^a(s) = f(sa)$ for any $a, s \in S$ and $f \in m(S)$).

The first to consider ELA semigroups (under different terminology) was T. Mitchell in [**13**]. His main and interesting result in [**13**] is that a semigroup S is ELA if and only if it has the common fixed point property on compacta (i.e. for each compact hausdorff space X and for each homomorphic representation S' of S as a semigroup (under functional composition) of continuous maps of X into itself, there is some x_0 in X such that $s'(x_0) = x_0$ for all s' in S'). This result is an analog of Day's generalisation of the Markoff Kakutani fixed point theorem [**4**].

Let $m(S)^*$ be the conjugate Banach space of $m(S)$. If $\phi \in m(S)^*$, let $(L_a\phi)(f) = \phi(f_a)$ for any $f \in m(S)$ and $a \in S$. Also let $1_a \in m(S)^*$ be defined by $1_a(f) = f(a)$ for $f \in m(S)$ and $a \in S$. Elements of $\{1_a; a \in S\}$ are called point measures. $\beta(S) \subset m(S)^*$ denotes the set of multiplicative means. $\beta(S)$ becomes a semigroup, which contains S, under the convolution multiplication: $\phi \odot \psi(f) = \phi(g)$ where $g(s) = \psi(f_s)$ for $s \in S$.

Define $r_a, l_a: m(S) \to m(S)$ by $r_a f = f^a$, $l_a f = f_a$ for $a \in S$. If $f \in m(S)$, denote by $K(f)$ the set of reals c for which there is some net in $\{r_a f; a \in S\}$ which converges pointwise to the constant function $c \cdot 1$ ($1 \in m(S)$ is the constant one function on S). S is *extremely right stationary* if $K(f) \neq \emptyset$ for each $f \in m(S)$. (Compare with Mitchell [**12**, p. 246]).

Let $A \subset m(S)$ be a uniformly closed left invariant (i.e., $f_s \in A$ for any $s \in S$, if $f \in A$) subalgebra with $1 \in A$. Denote by H_A the ideal of all $h \in A$ such that $h = \sum_1^n f_j(g_j - l_{a_j} g_j)$ for some $f_j, g_j \in A$ and some $a_j \in S$, $1 \leqq j \leqq n$ where $n = 1, 2, \cdots$. K_A will denote the linear sub-

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