EXTREMAL PROBLEMS IN THE CLASS OF CLOSE-TO-CONVEX FUNCTIONS[1]

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1. **Introduction.** A function $f(z)$, analytic in the open unit disc in the complex z-plane (denoted by D) is said to be *close-to-convex* if there exists a convex univalent function $\phi(z)$ such that $\operatorname{Re}(f'(z)/\phi'(z)) > 0$ for all z in D. In what follows, there is no loss of generality if we assume $f(z)$ to be normalized i.e., $f(0)=0$ and $f'(0)=1$. We can also assume that $\phi(0)=0$ and $|\phi'(0)|=1$. We denote the class of normalized close-to-convex functions by K. It is well known that K is a proper subclass of S—the family of normalized univalent functions in D (see [**3**]).

In this paper we announce the solutions to two general extremal problems within the class K and, as an application, we announce the rotation theorem for the class K. In the process of solving these extremal problems for the class K, the solutions to these extremal problems for several subclasses of K are found. Some of these solutions are known; we announce the results which do not appear to be known.

2. **Results for close-to-convex functions.** The first problem under consideration is a general coefficient problem. We have the following coefficient theorem for K.

THEOREM 1. *Let $F(z_2, \cdots, z_n)$ be any function having continuous derivatives in each of the $n-1$ variables $z_2, \cdots, z_n$. To each function $f(z)=z+a_2z^2+\cdots$ in K associate the number $\operatorname{Re}\{F(a_2, \cdots, a_n)\}$. Then any function $f(z)$ in K which maximizes $\operatorname{Re}\{F(a_2, \cdots, a_n)\}$ over the class K must be of the form*

$$f'(z) = \frac{e^{i\gamma}}{\prod_{j=1}^{M}(1 - ze^{i\alpha_j})^{\mu_j}} \sum_{k=1}^{N} \eta_k \frac{e^{-i\gamma} + ze^{i(\beta_k+\gamma)}}{1 - ze^{i\beta_k}}$$

where

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