THE INDEPENDENCE OF GAME THEORY OF UTILITY THEORY

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A theory of noncooperative and cooperative games, that parallels the classical theory [1], [2] but makes no use of utility theory, is outlined in this note.

1. **Games in normal form.** The following definition seems suitable for our purpose (see, e.g., [3, § 5]).

DEFINITION 1.1. *An n-person game* (in *normal form*) is a system $G=\{N;\ S^1, \cdots, S^n;\ X^1, \cdots, X^n;\ R^1, \cdots, R^n;\ H^1, \cdots, H^n\}$, where:

(1.1) N is a set of n members (the *players* of G), and for each $i \in N$:
(1.2) S^i is a nonempty set (the set of *strategies* of player i).
(1.3) X^i is a nonempty set (the set of *outcomes* for player i).
(1.4) $R^i \subset X^i \times X^i$ (the *preference relation* of player i).
(1.5) H^i is a function whose domain is the set $S = S^1 \times \cdots \times S^n$ and whose range is X^i (the *payoff function* of player i).

If $\bar{s} \in S$, $\bar{s} = (\bar{s}^1, \cdots, \bar{s}^n)$, and $s^i \in S^i$, we denote:

$$\bar{s} \mid s^i = (\bar{s}^1, \cdots, \bar{s}^{i-1}, s^i, \bar{s}^{i+1}, \cdots, \bar{s}^n) \tag{1.6}$$

DEFINITION 1.2. Let $G=\{N;\ S^1, \cdots, S^n;\ X^1, \cdots, X^n;\ R^1, \cdots, R^n;\ H^1, \cdots, H^n\}$ be an n-person game. $\bar{s} \in S$ is an *equilibrium point* for G if for each $i \in N$:

$$(H^i(\bar{s} \mid s^i), H^i(\bar{s})) \notin R^i, \quad \text{for all } s^i \in S^i. \tag{1.7}$$

This is Nash's definition [2] adjusted to our case.

2. **Finite, noncooperative games.** Let

$$G = \{N; S^1, \cdots, S^n; X^1, \cdots, X^n; R^1, \cdots, R^n; H^1, \cdots, H^n\}$$

be an n-person game. G is *finite* if S^i is finite for all $i \in N$. The *mixed extension* of[1] G is the n-person game

$$\hat{G} = \{\hat{N}; \hat{S}^1, \cdots, \hat{S}^n; \hat{X}^1, \cdots, \hat{X}^n; \hat{R}^1, \cdots, \hat{R}^n; \hat{H}^1, \cdots, \hat{H}^n\},$$

where:

$$\hat{N} = N, \tag{2.1}$$

[1] In what follows we assume that G is finite.