HIGHER-DIMENSIONAL SLICE KNOTS

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The purpose of this paper is to demonstrate the existence of higher-dimensional smooth slice (or 0-concordant) knots of spheres in spheres with generalized Alexander polynomials which are not symmetric and which do not factorize. In particular, this provides a negative answer to questions B and C of Hirsch and Neuwirth [1, Part II]. The method is then extended to provide a generalization of the results of Levine [2].

1. **Algebraic theory.** Consider an Abelian group A which is finitely generated as a module over the group ring JZ of the infinite cyclic group $Z(t)$ (generated by t). An $m\times n$ matrix $M=(m_{ij}(t))$ whose entries are polynomials in t (integer coefficients) is said to present A as a module if there exists an exact sequence of JZ modules

$$F_2 \xrightarrow{d_2} F_1 \rightarrow A \rightarrow 0$$

where F_1 and F_2 are free JZ modules on $(x_1, \cdots, x_n)$ and $(r_1, \cdots, r_m)$ respectively, and $d_2(r_i) = \sum_{j=1}^{n} m_{ij}(t)x_j$. See [3].

2. **Generalized Alexander polynomials.** A smooth n-knot is a smooth sphere pair (S^{n+2}, S^n). If $\pi_1(S^{n+2}-S^n)=G$, and G' = commutator subgroup of G, then the universal Abelian covering space $\tilde{X}$ of the knot complement $S^{n+2}-S^n$ is the regular covering space corresponding to G'. That is, $\pi_1(\tilde{X})=G'$, and the group of covering translations is the Abelian Group $G/G'=Z(t)$. The chain groups of $\tilde{X}$, and hence the homology groups $H_j(\tilde{X})$ are finitely generated as modules over $JG/G'=JZ(t)$, and have presentation matrices M_j, for all j (See [1]). In the terminology of [3], if $\epsilon_{1(j)}$ is the 1st elementary ideal of M_j and $\epsilon_{1(j)}$ is a principal ideal, then define the jth dimensional Alexander polynomial $\Delta_j(t)$ = generator of $\epsilon_{1(j)}$. If M_j is square, then $\Delta_j(t) = |M_j|$, the determinant of M_j. When $j=1$, the polynomial is the usual Alexander polynomial [4, p. 353]. Also, one can easily define a whole sequence of generalized Alexander polynomials in dimension j, each one corresponding to a higher elementary ideal of the presentation matrix M_j (See [2].).

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