ON CENTRAL TOPOLOGICAL GROUPS

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Introduction. Let G be a locally compact group and Z its center. We shall be concerned with the class $[Z]$ of all locally compact groups G such that G/Z is compact. In studying $[Z]$-groups the chief goal we have in mind is to obtain a natural generalization of the theory of compact groups and that of locally compact abelian groups broad enough to be nontrivial and which enables one to extend virtually all the important results pertaining to the aforementioned theories; this is done for the structure and representation theory. (For the latter see "Representation Theory of Central Topological Groups," p. 831 of this Bulletin).

1. **Extension theorems.** An interesting and useful feature of $[Z]$-groups is the extendibility of certain vector-valued homomorphisms defined on subgroups to crossed homomorphisms on the whole group.

Let G be a locally compact group, ρ a continuous finite-dimensional real representation of G on V and N a closed normal subgroup of G. As usual, a continuous function $\psi: G \to V$ is called a crossed homomorphism if $\psi(g_1g_2) = \rho(g_1)(\psi(g_2)) + \psi(g_1)$, for all g_1, g_2 in G, and a continuous function $\phi: N \to V$ is called G-invariant if $\phi(g)(\rho(x)) = \phi(g \times g^{-1})$, for all x in N, g in G.

THEOREM 1.1. *Let G, N, ρ, V be as above and suppose $G \in [Z]$. If ρ is a G-invariant homomorphism on N mapping into V^N, the N-fixed part of V, and either N or Z is open in G then ϕ extends to a crossed homomorphism ψ on G.*

The proof of this theorem makes use of the extendibility of continuous vector-valued homomorphisms defined on subgroups of locally compact abelian groups as well as the following well-known lemma due to P. Cartier [1], [5].

LEMMA 1.2. *Let G, N, ρ, V be as above. If G/N is compact and ϕ is a G-invariant homomorphism on N taking values in V^N then ϕ extends to a crossed homomorphism $\psi: G \to V$.*

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