WIENER'S CONTRIBUTIONS TO GENERALIZED HARMONIC ANALYSIS, PREDICTION THEORY AND FILTER THEORY

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0. **Prologue.** The strong cohesive forces permeating the work of Norbert Wiener complicate the task of surveying his contributions to specific areas. Where is one to begin and where to end? In the realm of prediction, for instance, Wiener's book [**TS**][1] stands out as his first major contribution. But an important part of this book concerns the synthesis of predictors, for which as Kakutani remarked (**32**): "The theory of generalized harmonic analysis developed by the author some 20 years ago is exactly the right tool . . . ." Now the latter theory, given in the memoir [**GHA**] of 1930, was itself the culmination of researches begun in 1924, which were motivated by even earlier investigations in the theory of Brownian motion. It would seem that a thorough review of Wiener's work in prediction should start from about the year 1919 when he looked at the Charles River from his office at M.I.T. and began to wonder whether the Lebesgue integral was the right tool for the analysis of the undulating water surface. Such a review would be beyond the abilities of this writer, even if he were granted the necessary space.

In this review we shall first survey those aspects of Wiener's great memoir [**GHA**] which bear on his later work on prediction and filtering (I). We shall then describe briefly how the mathematical activity of the thirties influenced his thought (II). Next we shall discuss Wiener's general theory of nonlinear prediction (III). From this we shall turn to his many contributions to linear prediction and filtering theory (IV). Lastly we shall dwell on his theory of filters (V).

I. Generalized harmonic analysis (1930)

1. **White light: the need for generalizing harmonic analysis.** The optical origins of Wiener's work are best expounded from the standpoint of the electromagnetic theory of light. At a fixed point r in a medium traversed by light the direction of propagation at instant t is given by the Poynting vector $P(t) = E(t) \times H(t)$, where $E(t)$, $H(t)$

[1] The bold-faced numbers in brackets refer to the numbered references in the Bibliography of Norbert Wiener. The bold-faced letters in brackets and numbers in parentheses refer to the References at the end of this article.