WIENER'S WORK IN PROBABILITY THEORY

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In 1918 and 1919 Daniell's papers on what is now called the Daniell integral appeared. As an application he constructed the most general finite measure of Borel sets in Euclidean space of countable dimensionality. In 1933 Kolmogorov, in his formalization of probability as measure theory, rediscovered the Daniell result in constructing measures on Euclidean space of arbitrary dimensionality. After 1933 probability to a mathematician was no longer merely an extra-mathematical source of interesting problems in analysis with colorful interpretations, but was now a normal part of mathematics whose historical development was commemorated by the use of peculiar names (random variable, expectation, . . .) for commonplace mathematical concepts (measurable function, integral, . . .).

Wiener was an exception in his probability work in that he almost never used the classical nomenclature, and in fact usually even avoided using the standard classical results and conventions, some of which would have simplified and clarified his work. He came into probability from analysis and made no concession. If his work had been less forbiddingly formal, it might have had even more influence.

In a series of papers beginning with **[12]*** Wiener undertook a mathematical analysis of Brownian motion. It was accepted that Brownian paths were governed by probabilistic laws, and it seemed plausible that the paths were continuous. The problem was to construct and analyze a rigorous mathematical model. More than a decade before Kolmogorov's formalization of probability Wiener constructed a mathematical model of Brownian motion in which the basic probabilities were the values of a measure defined on subsets of a space of continuous functions. This measure has since been commonly called "Wiener measure." Fixing an origin in time and a direction in space, let $x(t)$ be the component in the specified direction of the displacement by time t of a Brownian particle. Then $x(0)=0$. For technical reasons it was advantageous to restrict t to a compact interval. Thus Wiener was led to consider the space C of continuous functions on $[0, 1]$, vanishing at 0, and to define a measure of subsets of C (based on the Daniell integral). The probability of any property of the displacement function was associated with the measure of the subset of C having this property. The Wiener measure of subsets of C

* The bold-faced numbers in brackets refer to the numbered references in the Bibliography of Norbert Wiener.