NORBERT WIENER AND POTENTIAL THEORY

BY M. BRELOT

1. Wiener did not work in potential theory for very long (only about two years around 1924), but that was enough to bring about in this field (the so-called classical potential theory), as in many others, some fundamental contributions: a definitive form of the generalized solution of the Dirichlet problem for a continuous given boundary function, the notion of capacity for general compact sets and the famous Wiener criterion of regularity.

2. Already in 1923, he wrote with Phillips a paper **[28]*** on *Nets and the Dirichlet problem* where this problem was solved for a polycubic domain then for domains with smooth boundaries by a limit process from a problem for functions on a discrete net and a mean condition. This idea of using linear equations for a preliminary problem relative to finite differences, which is now a basic tool with computers for partial differential equations was not common forty years ago; I know only the previous example of Le Roux on harmonic functions in R^2 (J. Math. Pures Appl., 1914).

3. In pure potential theory, the first fundamental paper of Wiener **[24]** *Certain notions in potential theory* in January 1924 gave and studied a precise definition of a generalized solution and the first definition of capacity for an arbitrary compact set.

For a long time, it was known that the classical Dirichlet problem does not always have a solution (case of an isolated boundary point of Zaremba, spine of Lebesgue) and there appeared more or less clearly the need to define a suitable generalized solution which always exists that would be later studied at the boundary; such a harmonic function corresponding to the given boundary function was in evidence in various methods, where further restrictions on the boundary allowed to show that it was actually a solution (Poincaré, Zaremba, Lebesgue, Bouligand, Kellogg . . .). But in a clearer and more striking way than the others, Wiener introduced for a bounded open set (and for a similar "exterior" problem) a precise generalized solution, *that he studied further without restrictions*: it was the limit of the classical solution for an increasing sequence of regular open sets $\Omega_n \subset \Omega$ ($\bigcup \Omega_n = \Omega$) ("regular" means that there is always a solution for the classical Dirichlet problem) and a boundary function, given as

* The bold-faced numbers in brackets refer to the numbered references in the Bibliography of Norbert Wiener.