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PRINCETON UNIVERSITY AND

UNIVERSITY OF CALIFORNIA, BERKELEY

WEAK LEVI CONDITIONS IN SEVERAL COMPLEX VARIABLES¹

BY AVNER FRIEDMAN

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1. **Introduction.** Let $\Omega = \{z; z \in \Omega_0, \rho(z) < 0\}$ be a bounded domain in \mathbb{C}^n , where $\rho \in C^2(\Omega_0)$, Ω_0 a neighborhood of Ω , and let $\text{grad } \rho \neq 0$ on $\partial\Omega$. As is well known, if Ω is a domain of holomorphy then for any $x^0 \in \partial\Omega$,

$$(1) \quad L(\rho(x^0), w) \equiv \sum_{j,k=1}^n \frac{\partial^2 \rho(x^0)}{\partial z_j \partial \bar{z}_k} w_j \bar{w}_k \geq 0 \quad \text{whenever} \quad \sum_{j=1}^n \frac{\partial \rho(x^0)}{\partial z_j} w_j = 0,$$

and, if (1) holds with strict inequality (for $w \neq 0$) then Ω is a domain of holomorphy. (1) is called the *Levi condition* (LC) and, in case of strict inequality, the *strict LC*. One of the consequences of the present work is that the above statement remains true if the assumption $\rho \in C^2$ is replaced by $\rho \in H^{2,\infty}$ (see §2).

In what follows Ω is always given by ρ as above, where $\rho \in C^1(\Omega_0)$, $\text{grad } \rho \neq 0$ on $\partial\Omega$.

2. **Definitions.** If ρ has second weak derivatives which belong to $L^p(\Omega_0)$ ($1 < p < \infty$) then we say that Ω and ρ belong to $H^{2,p}$. Actually we shall only need the derivatives $\partial^2 \rho / \partial z_j \partial \bar{z}_k$ to belong to L^p , but then

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