A JORDAN DECOMPOSITION FOR OPERATORS IN BANACH SPACE

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Operators T with real spectrum in finite dimensional complex Euclidian space may be characterized by the property

$$(1) \qquad |e^{itT}| = O(|t|^k), \qquad t \text{ real}.$$

Our result is a Jordan decomposition theorem for operators T in reflexive Banach space which satisfy (1) and whose spectrum (which is real because of (1)) has linear Lebesgue measure zero.

1. **The Jordan manifold.** Let X be a complex Banach space; denote by $B(X)$ the Banach algebra of all bounded linear operators acting on X. For $m=0, 1, 2, \cdots, C^m$ is the topological algebra of all complex valued functions on the real line R with continuous derivatives up to the order m, with pointwise operations and with the topology of uniform convergence on every compact set of all such derivatives. Fix $T\in B(X)$. Following [3], we say that T is of class C^m if there exists a C^m-operational calculus for T, i.e., a continuous representation $f\to T(f)$ of C^m into $B(X)$ such that $T(1)=I$, $T(f)=T$ if $f(t)\equiv t$, and $T(\cdot)$ has compact support. The latter is then equal to the spectrum of T, $\sigma(T)$. It is known that if T satisfies (1), then it is of class C^m for $m\geqq k+2$ and has real spectrum (cf. Lemma 2.11 in [3]).

From now on, let $T\in B(X)$ satisfy (1), and let $T(\cdot)$ be the (unique) C^m-operational calculus for T, for m fixed $\geqq k+2$. We write:

1. $|f|_{m,T}=\sum_{j\leq m}\max_{\sigma(T)}|f^{(j)}|/j!$, $f\in C^m$;
2. $|x|_{m,T}=\sup\{|T(f)x|; f\in C^m, |f|_{m,T}\leqq 1\}$, $x\in X$;
3. $D_m=\{x\in X; |x|_{m,T}<\infty\}$;
4. $D=\bigcup_{m\geq k+2} D_m$.

We call D *the Jordan manifold for* T. It is an invariant linear manifold for any $V\in B(X)$ which commutes with T. If $\sigma(T)$ is a finite union of points and closed intervals, then there exists an $m\geqq k+2$ such that $D=D_m=X$. This is true for $m=k+2$ if $\sigma(T)$ is a finite point set. It follows in particular that D_{k+2} contains every finite dimensional invariant subspace for T, hence all the eigenvectors of T. It is also true that D contains all the root vectors for T, and is therefore dense in X if the root vectors are fundamental in X.

THEOREM 1. *Suppose that all nonzero points of* $\sigma(T)$ *are isolated.*