THE GALOIS THEORY OF INFINITE PURELY INSEPARABLE EXTENSIONS

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Introduction. Given a field K of characteristic $p\neq 0$, denote by $\mathrm{Der}(K)$ the set of all derivations of K. Then $\mathrm{Der}(K)$ is a vector space over K, and a Lie subring of the ring of additive endomorphisms of K. Moreover, $\mathrm{Der}(K)$ is closed under pth powers. A Lie ring satisfying this additional closure property is called a *restricted* Lie ring. Take any subfield F of K such that K over F is of exponent one, i.e., $K^p\subset F$. Denote by $\mathrm{Der}(K/F)$ the set of all derivations of K which vanish on F. Then $\mathrm{Der}(K/F)$ is a vector subspace and restricted Lie subring of $\mathrm{Der}(K)$. On the other hand, take a restricted Lie subring D of $\mathrm{Der}(K)$ which is also a vector subspace over K. Let $\Phi(D) = \{x\in K \mid \lambda(x)=0 \text{ for every } \lambda\in D\}$. Then $\Phi(D)$ is a subfield of K such that K over $\Phi(D)$ is of exponent one. This gives a one-to-one correspondence between subfields F of K over which K is *finite* and of exponent one, and restricted Lie subrings of *finite* dimension over K (cf. [1] and [2]). The purpose of this note is to extend this Galois correspondence to the infinite dimensional case. The first half of the correspondence is valid regardless of the dimension of K over F, i.e., $\Phi(\mathrm{Der}(K/F))=F$ if $K^p\subset F$ [1, p. 183]. However, to establish the second half of the correspondence, one must put a stronger condition on the vector subspace of $\mathrm{Der}(K)$, namely, that of p-convexity.

p-convexity. Let us fix a field K of characteristic $p\neq 0$. Since we shall only consider subfields F for which $K^p\subset F$, we should *designate* K^p as our base field. For every $x\in K$, let H_x denote the set of all λ in $\mathrm{Der}(K)$ such that $\lambda(x)=0$. H_x may be regarded as a "distinguished" hyperplane in $\mathrm{Der}(K)$. We call a subspace V of $\mathrm{Der}(K)$ *p-convex* if $V=\bigcap(V+H_x)$, the intersection being taken over all $x\in K$.

THEOREM 1. *Let V be a vector subspace of* $\mathrm{Der}(K)$ *which is p-convex, and let $F=\Phi(V)$. Then* $\mathrm{Der}(K/F)=V$, *which implies that every p-convex subspace of* $\mathrm{Der}(K)$ *is automatically a restricted Lie subring of* $\mathrm{Der}(K)$. *Conversely, if F is a subfield of K containing K^p, then* $\mathrm{Der}(K/F)$ *is p-convex.*

PROOF. Let $\lambda\in\mathrm{Der}(K/F)$. Take any element x of K. If x is in F, then $\lambda(x)=0=\mu(x)$ for any $\mu\in V$. Suppose that x is not in F. Let $E_x=K^p(x)$. Then V restricted to E_x must be a nonzero vector subspace of $D(E_x, K)$, the set of all derivations of E_x into K. Denote by