RESEARCH PROBLEMS

18. Fred Gross: *Function theory*

Let S be an arbitrary region. Does there exist a transcendental meromorphic function with the property that the pre-image $f^{-1}(S)$ is of finite measure? (Received July 16, 1965.)

19. Fred Gross: *Function theory*

In Volume 4 of the Michigan Mathematical Journal Paul Erdös asked the following question:

If A and B are two denumerable dense sets, does there exist an entire function which maps A onto B?

This problem is quite difficult. One can ask, however, a simpler question.

Do there exist two dense denumerable sets A and B, such that, $f(z) \in A$ if and only if $z \in B$ implies that $f(z)$ is linear? More generally, do there exist two linearly independent functions $f(z)$ and $g(z)$ such that $f(z) \in A$ if and only if $g(z) \in B$. (Received July 16, 1965.)

20. Albert A. Mullin: *Stochastic number-theory*

The probability that a random natural number has the prime factor p is $1/p$. Hence the probability that two random natural numbers have the common prime factor p is $1/p^2$. Thus, the probability that two random natural numbers have no common prime factor (i.e., the probability that they are relatively prime) is $\prod_p (1-1/p^2) = 1/\zeta(2) = 6/\pi^2$, by Euler's Identity (an *analytic* version of unique factorization. What is the probability that two random natural numbers satisfy the condition that their *mosaics* have no prime in common? Clearly this measure is strictly positive but it is bounded above by $6/\pi^2$. To what extent is statistical *dependence* crucial to the argument? What is the probability that a random natural number has only odd primes in its mosaic? (Received July 16, 1965.)

21. Richard Bellman: *Differential equations*

The equation $u'(t) = (a+bu(t_1))u(t)$, $u(0)=c$, valid for $0 \leqq t \leqq t_2$, with $t_2 > t_1$, shows that an equation of the form $u'(t) = g(u(t), u(t_1), u(t_2), \cdots, u(t_N))$, $u(0)=c$, $0<t_1<t_2< \cdots <t_N$, can have an infinite number of complex solutions and more than one real solution. What additional conditions on the solution ensure uniqueness of a real solution? (Received July 16, 1965.)