ON THE THEORY OF RANDOM SEARCH

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Introduction. The problems of search dealt with in this paper can be described by the following simple model. Let S_n be a *finite* set having $n \geqq 2$ distinguishable elements—called points—and suppose that we want to find an unknown point x of the set S_n; the set S_n itself is supposed to be known to us. Let us suppose further that it is not possible to observe x directly, however we may choose some functions $f_1, f_2, \cdots, f_N$ from a given set F of functions defined on S_n, and observe the values $f_1(x), f_2(x), \cdots, f_N(x)$ taken on by these functions at the unknown point x. Of course if F would contain a function f which takes on different values at different points, a single observation of this function would be sufficient. We suppose however that all functions f belonging to the class F are such that the number of different values taken on by f is much smaller than n. (We shall be especially interested in the case when each $f \in F$ takes on only the two values 0 and 1 and n is a large number.) In such a case of course it is necessary to observe the value of a large number of functions f at the point x. Each such observation gives us only partial information on x (namely it specifies a subset A of S_n to which x must belong), but after making a fairly large number of such observations the information obtained accumulates and enables us to determine x. We want to find x by a not too large number of observations. We may e.g. suppose that each observation is connected with a certain cost (or that it requires a definite amount of time) and we want to keep the cost (or duration) of the whole procedure of search relatively low. We shall call a method for the successive choice of the functions $f_1, \cdots, f_N$, which leads in the end to the determination of the unknown x, a *strategy of search*. Obviously one usually tries to choose a strategy with N (the number of functions to be observed) as small as possible. Of two search procedures the one which has a smaller (average) duration is the better one, however there may be other requirements. For instance a simple strategy which can e.g. be easily programmed on a computer is usually preferable to a complicated strategy. If A and B are two strategies such that A requires (in the average) the observation of a somewhat smaller number of functions than B (i.e. A is "better" than B) but the effective carrying out of A

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