SOLVABILITY OF THE FIRST COUSIN PROBLEM AND VANISHING OF HIGHER COHOMOLOGY GROUPS FOR DOMAINS WHICH ARE NOT DOMAINS OF HOLOMORPHY[1]

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This work is a sequel to [1]: In [1] we considered the vanishing of the first cohomology groups with coefficients in $\mathcal{O}$, $\mathcal{O}^*$ for sets $X\backslash A$ whereas in the present work we consider the same question for higher cohomology; at the same time we obtain some additional results for the first Cousin problem. As in [1] we take $n \geqq 3$.

Scheja [3] proved that if X is an open set in $\mathbf{C}^n$ and A is an analytic closed subset of X of dimension $\leqq n-q-2$, then the natural homomorphism

$$(1) \qquad H^q(X, \mathcal{O}) \to H^q(X\backslash A, \mathcal{O})$$

is bijective. We shall prove:

THEOREM 1. *Let A be a closed bounded generalized polydisc in an open set X of $\mathbf{C}^n$. Then the natural homomorphism* (1) *is bijective for any* $1 \leqq q \leqq n-2$.

PROOF. Set $A = L_1 \times \cdots \times L_n$ and let $K = K_1 \times \cdots \times K_n$ be an open generalized polydisc with $A \subset K \subset \bar{K} \subset X$. Set $L' = L_2 \times \cdots \times L_n$, $K' = K_2 \times \cdots \times K_n$, $G_0 = (K_1 \backslash L_1) \times K'$, $G_1 = K_1 \times (K' \backslash L')$, $G = G_0 \cup G_1$. By a straightforward generalization of [3, Hilfsatz] one gets $H^q(G, \mathcal{O}) = 0$. We now introduce a covering $U = \{U_i\}$ of $X\backslash A$ where all the U_i are domains with $H^q(U_i, \mathcal{O}) = 0$ and precisely $q+1$ of them, say $U_{i_0}, \cdots, U_{i_q}$, coincide with G. By Leray's theorem [2], the canonical homomorphism

$$(2) \qquad H^q(N(U), \mathcal{O}) \to H^q(X\backslash A, \mathcal{O})$$

(where $N(U)$ is the nerve of U) is bijective.

We next introduce a covering $U' = \{U_i'\}$ of X where $U'_{i_0} = \cdots = U'_{i_q} = K_1 \times K'$ and $U_i' = U_i$ for all other indices i. Again, the canonical map

$$(3) \qquad H^q(N(U'), \mathcal{O}) \to H^q(X, \mathcal{O})$$

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