AN ANNIHILATOR ALGEBRA WHICH IS NOT DUAL[1]

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1. **Introduction.** The purpose of this note is to give an example of an annihilator algebra which is not dual; no other such example has been published. Here we construct a semi-simple, normed, annihilator algebra which has a closed two-sided ideal which is not an annihilator algebra. Every dual algebra is an annihilator algebra by definition, and every closed ideal in a semi-simple dual algebra is a dual algebra by a theorem of Kaplansky ([**2**, Theorem 2, p. 690] or (iii) in the text of this note). Noting these facts, it follows that the example we construct is not a dual algebra.

Whether every closed two-sided ideal in an annihilator algebra was necessarily an annihilator algebra had been a question of long standing.

The example given here is a normed algebra. The algebra is a Q-algebra (see [**4**, p. 373]), but not, however, a Banach algebra in the given norm. Therefore these questions remain open for the special case of a Banach algebra.

2. **The example.** Let l^p be the algebra of p-summable complex sequences with multiplication performed coordinate-wise. Set $A_1 = l^1$, $A_2 = l^2$ and $A = A_1 \oplus A_2$ (the direct sum of A_1 and A_2). For $x \in A$, we shall write $x = (x_1, x_2)$, where $x_1 \in A_1$, $x_2 \in A_2$. $x_1(i)$ and $x_2(i)$ will denote the ith coordinate of x_1 and x_2 in l^1 and l^2, respectively.

We shall define a norm on A such that A is an annihilator algebra, but not dual, in the topology of this norm.

First we define, for $x \in A$,

$$p(x) = \left(\sum_{i=1}^{\infty} |x_1(i)|^2 + \sum_{i=1}^{\infty} |x_2(i)|^2 \right)^{1/2}.$$

Note that $p(x)$ is a norm on A.

Secondly, since l^1 is properly contained in l^2, we may choose a nonzero linear functional F on l^2 such that $F(x) = 0$ for $x \in l^1$. Furthermore, since $(l^2)^2 = l^1$, F is zero on $(l^2)^2$. Now we define, for $x \in A$, $x = (x_1, x_2)$,

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