"RECURSIVE ISOMORPHISM" AND EFFECTIVELY EXTENSIBLE THEORIES[1]

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This paper is concerned with the following problem: to what extent can we obtain a meaningful classification of mathematically interesting formal theories by virtue of their recursive properties? Myhill's results on the "recursive isomorphism" of creative theories [2] indicate that it may not be sufficient to identify a theory with its set of theorems. To what extent can a "recursive isomorphism" preserving the deductive structure of the theories provide a meaningful classification? Accordingly we will concern ourselves with how a theory is presented in terms of axioms and rules of inference (see 2 below).

DEFINITIONS. 1. A *theory* $\mathfrak{T}_i$ is an ordered triple $\langle W_i, T_i, R_i\rangle$, where W_i is a recursive set and T_i and R_i are recursively enumerable sets satisfying $T_i \subseteq W_i$ and $R_i \subseteq W_i$. Theory $\mathfrak{T}_i$ is *consistent* if $T_i \cap R_i = \emptyset$.

Intuitively, W_i stands for the set (of Gödel numbers) of all statements, T_i the set of theorems and R_i the set of refutable statements. Thus $W_i - (T_i \cup R_i)$ represents the set of undecidable statements. Note that all theories considered in this paper are axiomatizable. They are also assumed to be consistent.

2. We now define a *presentation.* If $\mathfrak{T}$ possesses negation the following definition suffices. A *presentation of a theory* is an ordered pair $\langle \alpha, \mathbf{R}\rangle$, α a recursively enumerable set (the set of axioms) and $\mathbf{R}$ a recursively enumerable sequence of recursively enumerable relations (the rules of inference). Furthermore, $\phi \in T$ if and only if ϕ can be obtained from a finite number of members of α by a finite number of applications of finitely many members of $\mathbf{R}$. Note that membership in R is determined by membership in T.

If $\mathfrak{T}$ does not possess negation the above definition must be modified so that the presentation possesses a component which generates

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[2] Some of these results were presented to the Association of Symbolic Logic, April 21, 1964.