ON SIMULTANEOUS EXTENSION OF CONTINUOUS FUNCTIONS

BY H. H. CORSON[1] AND J. LINDENSTRAUSS[2]

Communicated by V. Klee, January 14, 1965

Let S be a compact Hausdorff space and let K be a closed subset of S. Denote by $C(S)$ (respectively, $C(K)$) the Banach space of all continuous real-valued functions on S (respectively, K) with the supremum norm. A bounded linear operator T from $C(K)$ to $C(S)$ is called a simultaneous extension (s.e.) operator if the restriction of Tf to K is equal to f for every $f \in C(K)$. Put

$$\eta(K, S) = \inf\{\|T\|;\ T \text{ is an s.e. operator from } K \text{ to } S\}.$$

($\eta(K, S) = \infty$ if there exists no s.e. operator from K to S.) Several authors (for example, Borsuk, Kakutani, Dugundji and Arens, cf. the expository paper [4] for references) have considered this notion of simultaneous extension of continuous functions. It is known that, if K is metrizable, then $\eta(K, S) = 1$ for every $S \supset K$ (cf. the recent paper [3] for a much stronger result), and examples of K and S for which $\eta(K, S) = \infty$ are known. As far as we know, in all examples considered thus far either one of these two extreme situations occurred.

In this note we find all the possible values of $\eta(K, S)$ for K the one-point compactification of an uncountable set (which is, in a sense, the simplest nonmetrizable compact Hausdorff space). The result we obtain is somewhat surprising and it indicates that the study of the behaviour of $\eta(K, S)$ for more general K may be of interest. We intend to consider this question as well as the more general question of extending maps into nonmetrizable compact convex sets in a future paper (cf. also Proposition 1 in this note).

THEOREM 1. *Let K be the one-point compactification of an uncountable set. Then, for every compact Hausdorff S containing K, $\eta(K, S)$ is either an odd integer or ∞. Conversely, for every integer n there is an $S_n \supset K$ such that $\eta(K, S_n) = 2n+1$ and there is also an $S_\infty \supset K$ such that $\eta(K, S_\infty) = \infty$.*

Denote by Σ the unit cell of $C(K)^*$. Σ consists of all measures μ on K with total variation $\|\mu\| \leqq 1$ and it is compact Hausdorff in

[1] Research fellow of the Alfred P. Sloan Foundation.

[2] Research supported in part by the National Science Foundation (NSF GP-378).