WEIGHTED ENTIRE FUNCTIONS

BY K. RAMAN UNNI[1]

Communicated by R. Boas, November 9, 1964

The aim of this note is to announce some results concerning the weighted entire function $z^\alpha f(z)$, where $\alpha \geqq 0$ and $f(z)$ is an entire function of exponential type. All these results are known to be true when $\alpha = 0$. Boas [2] proved that if $f(z)$ is an entire function of exponential type such that $f(x) \in L^p(-\infty, \infty)$, then $f'(x) \in L^p(-\infty, \infty)$. Plancherel and Pólya [6] showed that if $f(z)$ is an entire function of exponential type such that $f(x) \in L^p(-\infty, \infty)$, then $\sum_{n=-\infty}^{\infty} |f(n)|^p < \infty$; if the type of $f(z)$ is less than π, then the converse is also true. Harvey [4] proved a number of results concerning the pth mean values of an entire function of exponential type. In this note we generalize all these results to weighted entire functions. These results will appear with proofs soon.

Let $z = x + iy$, where x and y are real, denote the complex variable. Suppose $p > 0$, $\alpha \geqq 0$ and $f(z)$ is an entire function of exponential type. We set, for each fixed real number a,

$$M_T^{p,\alpha}[f(x + a + iy)] = \frac{1}{2T} \int_{-T}^{T} | x^\alpha f(x + a + iy) |^p \, dx.$$

If n is a positive integer, we set, for each fixed integer m,

$$N_n^{p,\alpha}[f(x + m)] = (2n + 1)^{-1} \sum_{k=-n}^{n} | k^\alpha f(k + m) |^p.$$

We define

$$M^{p,\alpha}[f(x + iy)] = \limsup_{T \to \infty} M_T^{p,\alpha}[f(x + iy)]$$

and

$$N^{p,\alpha}[f(x)] = \limsup_{n \to \infty} N_n^{p,\alpha}[f(x)].$$

Here $M^{p,\alpha}[f(x)]$ and $M^{p,\alpha}[f(x+iy)]$ are the weighted pth mean of $f(z)$ along the real axis and along a line parallel to the real axis, re-

[1] Presented to the Society, April 30, 1963. The contents of this paper form part of the author's dissertation at Northwestern University (August, 1963), written under the direction of Professor R. P. Boas, Jr. This research was partially supported by NSF Grant GP-314.