RESEARCH PROBLEMS

4. R. A. Hirschfeld: *Invariant subspaces.*

E is a complex locally convex vector space, in which every closed bounded subset is complete. Let $T: E \to E$ be a linear continuous operator with nonempty spectrum, possessing a continuous inverse $T^{-1}: E \to E$.

Assume the family $(T^n)_{n=-\infty}^{\infty}$ to be equicontinuous.

Is it true that there is an invariant closed nontrivial linear subspace for T? (For a Banach space the answer is yes.) (Received December 4, 1964.)

5. R. A. Hirschfeld: *Extension of nonlinear contractions.*

E and F are Banach spaces, F reflexive, D is a subset of E and $T: D \to F$ a nonlinear contraction, i.e., $\|Tx_1 - Tx_2\|_F \leqq \|x_1 - x_2\|_E$ whenever $x_1, x_2 \in D$.

Can T be extended to a contraction $\tilde{T}: E \to F$? (For $E = F =$ Hilbert space the answer is yes.) (Received December 4, 1964.)

6. Richard Bellman: *Factorization of linear differential operators modulo p.*

Let D represent the operator d/dx. Consider the factorization

$$D^2 + a_1(x)D + a_2(x) = (D + b_1(x))(D + b_2(x)),$$

where a_1, a_2, b_1, and b_2 are polynomials in x of degree less than p, a prime, and the equality is required to hold modulo p. What is the number of irreducible linear differential operators for the case where $a_1(x)$ and $a_2(x)$ are required, respectively, to have degrees m_1 and m_2? Generalize to the case of linear differential operators of the form $D^n + a_1(x)D^{n-1} + \cdots + a_n(x)$. (Received November 30, 1964.)

7. Richard Bellman: *Functional differential equations.*

Under what condition on the function $r(t) \geqq 0$ can one assert that all solutions of $u'(t) + au(t - r(t)) = 0$ approach zero as $t \to \infty$?

Under what conditions do all solutions of $u'(t) = au(t - r(t)) = \sin bt$ approach $c \sin bt$ as $t \to \infty$?

If all solutions of $u'(t) + au(t - r) = 0$ approach zero as $t \to \infty$, and if $|r(t) - r| \leqq \epsilon$ for $t \geqq 0$, do all solutions of $u'(t) + au(t - r(t)) \to 0$, as $t \to \infty$, for ϵ sufficiently small? (Received November 30, 1964.)