THE OBSTRUCTION TO THE LOCALIZABILITY OF A MEASURE SPACE[1]

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This paper, an outgrowth of the author's doctoral dissertation,[2] presents a necessary and sufficient condition, of a cohomological nature, for a measure space to be localizable in the sense of Segal.[3] In order to state the main theorem, we must fix some terminology and establish some notation.

1. **Definitions.**[4] A *measure space* (X, R, m) consists of a set X, a boolean ring R of subsets of X, and a finite, nonnegative, finitely additive measure m on R subject to the requirement:

$$\{E_n \in R \ (n = 1, 2, \cdots), E_n \cap E_k = \varnothing \ (n \neq k), \\ \Sigma_n m(E_n) < \infty, \ E = \bigcup_n E_n\} \Rightarrow \{E \in R \text{ and } m(E) = \sum_n m(E_n)\}.$$

If (X, R, m) is a measure space, a subset K of X is *measurable* if $K \cap E \in R$ whenever $E \in R$; it is *null* if it is measurable and $m(K \cap E) = 0$ whenever $E \in R$. The *measure ring* $\mathfrak{M}$ of the measure space (X, R, m) is the quotient of the (sigma ring of) measurable sets by the (sigma ideal of) null sets. A measure space is *localizable* if its measure ring is complete as a partially ordered set.

2. Let (X, R, m) be a measure space. Consistent use will be made of the following notation:

I: the ideal of sets $K \in R$ for which $m(K) = 0$;
M_1: the sigma ring of measurable sets;
X_R: the set $\bigcup R \in M_1$;
M: the principal ideal of M_1 determined by X_R;
N_1: the sigma ideal of null sets in M_1;
N: the sigma ideal of null sets in M, i.e., $M \cap N_1$;

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[2] F. E. J. Linton, *The functorial foundations of measure theory*, Columbia Univ., New York, 1963.

[3] I. E. Segal, *Equivalences of measure spaces*, Amer. J. Math. **73** (1951), 275–313.

[4] This is merely a restatement, for the convenience of the reader, of parts of Definitions 2.1, 2.2, 2.4, and 2.6 in the cited work of Segal. Incidentally, our rings need not have unit elements.