MULTIPLIERS OF FOURIER TRANSFORM IN A HALF-SPACE[1]

BY E. SHAMIR

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1. Let (x, y) denote points in R^n where $x=(x_1, \cdots, x_{n-1})$, $y=x_n$. Points of the dual space are denoted by (ξ, η). Let Y_+ be the characteristic function of the half space $R^n_+ = \{(x, y) \mid y \geqq 0\}$. Let $M(\xi, \eta)$ be an $m \times m$ matrix-valued function whose entries are homogeneous functions:

$$M_{ij}(\lambda\xi, \lambda\eta) = M_{ij}(\xi, \eta), \qquad \lambda > 0, 1 \leqq i, j \leqq m.$$

Assume further that $M(\xi, \eta)$ is continuous and nonsingular for $(\xi, \eta) \neq 0$. Consider the bounded operator $\boldsymbol{M}$ in the space $(L^2(R^n_+))^m$ (with the natural norm denoted by $\| \ \|$):

$$(1) \qquad \boldsymbol{M}u = Y_+\mathfrak{F}^{-1}[M(\xi, \eta)(\mathfrak{F}u)(\xi, \eta)], \qquad u \in (L^2(R^n_+))^m,$$

where $\mathfrak{F}$ ($\mathfrak{F}^{-1}$) denotes the direct (inverse) Fourier transform with respect to all variables. $\mathfrak{F}_y$ ($\mathfrak{F}_x$) will denote the transform with respect to y or x alone. The one-dimensional operator $\boldsymbol{M}_\xi$ is similarly defined in $(L^2(R^1_+))^m$ with the multiplier $M(\xi, \eta)$, ξ fixed:

$$(2) \qquad \boldsymbol{M}_\xi v = Y_+\mathfrak{F}_y^{-1}[M(\xi, \eta)(\mathfrak{F}_y v)(\eta)].$$

Our main results in this note are the following lemma and theorem.

LEMMA. *The estimate*

$$(3) \qquad \|u\| \leqq C\|\boldsymbol{M}u\|, \qquad u \in (L^2(R^n_+))^m$$

holds if and only if for all $|\xi| = 1$ (*uniformly*)

$$(4) \qquad \|v\| \leqq C\|\boldsymbol{M}_\xi v\|, \qquad v \in (L^2(R^1_+))^m.$$

For the scalar case $(m=1)$, we have

THEOREM. *Let* $M(\xi, \eta)$ *be a homogeneous function continuous and nonvanishing for* $(\xi, \eta) \neq 0$. *Let*

$$(5) \qquad -\frac{1}{2\pi}\int_{-\infty}^{\infty} d_\eta \arg M(\xi, \eta) = k + \theta, \qquad k \text{ integer}, -1/2 < \theta \leqq 1/2.$$

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