JORDAN ALGEBRAS OF SELF-ADJOINT OPERATORS[1]

BY DAVID M. TOPPING

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1. **Introduction.** The purpose of this note is to announce a *real* noncommutative (more precisely, nonassociative) generalization and counterpart to the theory of von Neumann algebras. The algebras in question are weakly closed Jordan algebras of self-adjoint (s.a.) operators, to be referred to below as JW-*algebras*.

The results obtained are remarkably parallel to the global von Neumann theory. Our principal contributions are the development of a theory of relative dimension, culminating in the Comparison Theorem (from which a variety of structural information is obtained) together with an example of a new factor phenomenon not occurring in the von Neumann theory. Details and proofs will be published in the Memoirs of the Society.

2. **Quadratic ideals and annihilators.** Let A be a JW-algebra and M any subset of A. The *annihilator* of M is the set $M^{\perp}=\{a\in A: ab=0$ for all $b\in M\}$ (ab denotes the ordinary operator product).

A *quadratic ideal* is a linear subspace I of A with $aba\in I$ whenever $a\in I$ and $b\in A$ (note that $aba=2a\circ(a\circ b)-a^2\circ b$, where $a\circ b=\frac{1}{2}(ab+ba)$). The *center* of A is the set $Z=\{z\in A: za=az$ for all $a\in A\}$.

THEOREM 1. *The annihilators in a* JW-*algebra A are precisely the weakly closed quadratic ideals, and are of the form $eAe=\{eae\colon a\in A\}$, where e is a projection in A. The projections form a complete orthomodular lattice (so A has a largest projection which we assume is the identity operator* 1). *The annihilator of a Jordan ideal has the form eAe with e central. For a projection $e\in A$, eAe is a Jordan ideal if and only if e is central. The annihilator of a central subset is a direct summand.*

As usual, we define the *central cover* $C(a)$ of $a\in A$ to be the smallest central projection e with $ea=a$. We call a *faithful* if $C(a)=1$.

COROLLARY 1. *The central cover $C(a)$ exists and is the unique central projection e for which $(a)^{\perp}=\{e\}^{\perp}$, where (a) is the principal Jordan ideal generated by a.*

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