CATEGORICAL ALGEBRA

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1. Introduction. Category, functor, and natural transformation of functors are notions of great generality—and consequent simplicity. They apply to many different parts of mathematics. Now 22 years old, they have recently attracted especially active interest in many quarters. This interest is probably a reflection of the very rapid current proliferation of mathematical ideas—a situation favoring and, indeed, almost requiring unifying notions such as those of category and functor.

This article, a revision of the notes used in the Colloquium Lectures of the American Mathematical Society for 1963, will summarize a number of the developments which use categories, with particular attention to the ubiquity of adjoint functors, the utility of abelian categories, a unified categorical treatment of types of algebras, relative homological algebra via adjoint functors, differential graded objects, and universal algebra via suitable "very small" categories. For some items of detail, I refer to my book *Homology* [77] and references in the style "Gertrude Stein [1929]" are to the bibliography there. Other references, in the usual style, are to the bibliography at the end, which is intended to cover recent literature of the subject in tolerable completeness.

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CHAPTER I. FUNCTORS AND ADJOINTS

2. Categories. Let $\mathbf{C}$ be a class of objects $A, B, C, \cdots$ together with a family of disjoint sets $\hom(A, B)$, one for each ordered pair A, B of objects. Write $f: A \to B$ for $f \in \hom(A, B)$, and call f a *map* or

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